Team 1185

What’s the largest payload that could be launched into orbit by slingshot?

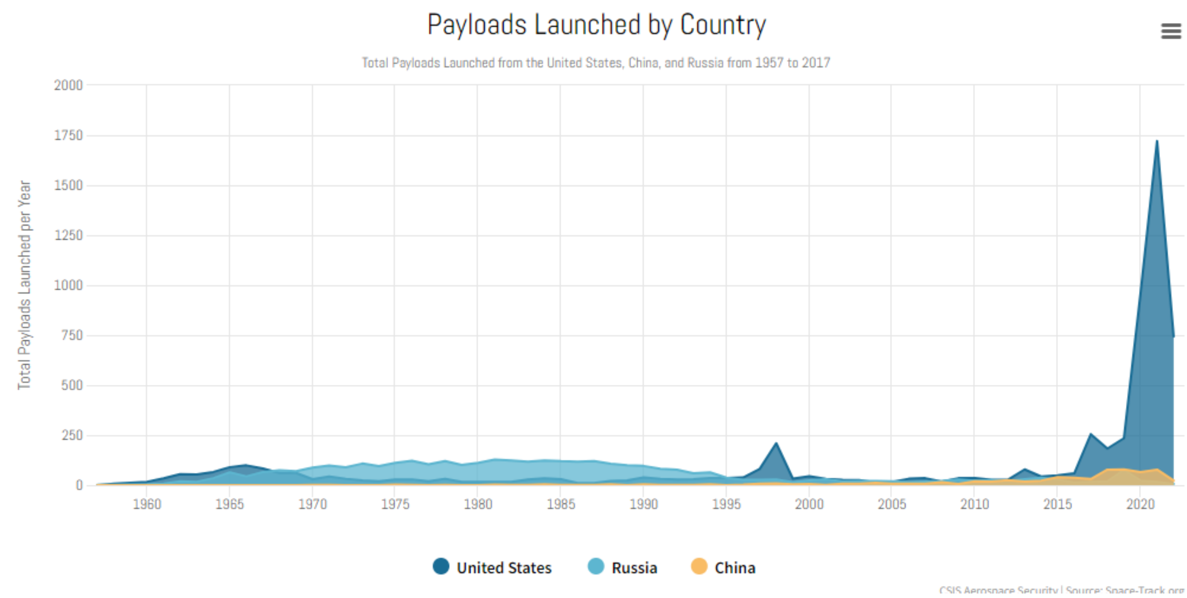
Abstract:

The purpose of this report was to calculate the largest (heaviest) payload that could be launched into orbit by a slingshot. We have decided to launch our payload from Mount Cook into low Earth orbit (LEO) using the SpinLaunch orbital system. This seemed to be the best environmental, economical and practical decision to achieve our goal of launching the largest payload into orbit by slingshot. Through mathematical modelling and the help of Newtonian mechanics, we found that the heaviest payload we could launch into orbit with our “slingshot” would be 106 kg, which is rounded to 3 significant figures.

Introduction:

As we progress into the 21st century, there has never been a time where space technology and exploration has been as accessible as it is today. The complexity of the technology that is out there in the world has significantly evolved which has allowed us to send more and more payloads into space each year. Over the past few years, there has undoubtedly been an increase in demand to launch payloads into space. To further support this point, Figure 1 below shows us that the USA alone launched approximately 1750 payloads into space in 2021 alone. However, this huge spike in the number of payloads launched does come with drastic environmental ramifications. Climate change has been a hot topic around the world over the past few years. People have been pushing for large businesses and corporations to change and become more aware of the emissions they release during the production of their goods. In the case of space exploration and payload launches, it is no surprise these satellites into orbit leave a large carbon footprint on our Earth. This has become a concern for people as it doesn’t help us reduce the carbon emissions we need to meet the Paris Agreement. SpinLaunch is the name of a California tech company who gained traction after signing with NASA. NASA will work with them to test their environmentally friendly method that innovates the way that we send payloads into space. They aim to use what has been labelled as a “**giant slingshot** [1] ” by the New York Post to release payloads into space. Their system is “**expected to require 70% less fuel than traditional satellite launches.**[2] ” This is as the mechanism to release the payload does not require a rocket, hence reducing the emissions released per launch.

Figure 1



CSIS Aerospace Security | Source: Space-Track.org

Definition & Interpretation of Question:

Due to the open nature of this question, we intend to keep our variables as practical as possible to ensure that we are able to answer the question realistically.

Although a slingshot is generally defined as a hand-held device which is able to shoot a projectile with an elastic band, we are straying away from this definition to ensure that we can realistically achieve our goal of sending a payload into orbit. For the purpose of this report, we will define a slingshot as an object which has a mechanism to “spin or wind up” and eventually release the stored energy at an instant. This definition still matches the general attributes of a hand-held slingshot, which stores elastic potential energy before transforming into kinetic energy. Although the question is open to interpretation, we prefer to not rely on a rocket to be our sole producer of energy as that would take away the slingshot aspect from the question.

We define “largest” to refer to the weight of the payload in kg. NASA describes a payload as “a valuable space craft, cargo, or people that need to be delivered into space.” However, as mentioned in the abstract, we will be utilising the SpinLaunch orbital system which means that humans will not be able to board the spacecraft or payload. This is because in terms of G forces, the system is 10,000 G’s, which is impossible for a human to withstand. To put this into context, if a 60kg person was on the payload, they would feel like they weighed 600,000kg. This is more than enough force to “tear and rip their muscles apart.” Furthermore, there is no chance that the orbital launch system will have the capability to send a satellite at the level of the ISS for example. The mass of the ISS is around 425,000 kg which far exceeds the realistic capabilities of the orbital system we intend to use. Therefore, in this report, we will define a payload as cargo which is to be delivered into space.

We have interpreted the question to be asking us for the heaviest payload in kilograms that would be able to exit and orbit the Earth with a launcher that has “slingshot” properties.

However, to achieve an accurate and realistic answer, we must follow some of the assumptions below.

Assumptions:

- Low Earth Orbit
- Circular Orbit
- Earth is a perfect sphere with no extra movement
- Force of gravity is constant at all heights
- Negligible air friction/resistance

Our first assumption that we made during our investigation was what kind of orbit we wanted to achieve. We decided on a Low Earth Orbit, or LEO for short, due to the practicality of our "slingshot" as the payload wouldn't be able to be launched into a solar orbit due to how much energy is provided by the centrifuge alone. Therefore we picked the minimum height for an LEO which was 160 km above the surface of the Earth to increase the chance of achieving an orbit. We also assumed that the orbital path of the payload would be perfectly circular, similar to that of the International Space Station, a craft in a LEO that travels at a circular orbit around the Earth and not an elliptical orbit. These were the assumptions we made about what it means for the payload to be in orbit, but more would have to be made on how we got it into one in the first place. One being that the Earth is a perfect sphere, which would increase the chance that the payload's orbit is in a circular path and the best model of a projectile motion. Another was that the force of gravity is constant at all heights, including at the site of launch and when in orbit. After figuring out the difference in gravitational forces at the two heights, there was only a 5% difference which we found to be negligible in the grand scheme of our investigation. We also did not take air resistance into account in our calculations as they would also not have a perceivable effect on our answers.

Choice of Slingshot & the Impact of Launch Location:

First we considered how a normal handheld slingshot works, how it uses the elastic band connected to two posts, to shoot small projectiles at high velocities and if this could be scaled to large proportions. However, we encountered the problem of how practical this larger slingshot could be. We thought about how large the elastic band would have to be and if it would be feasible, how much we would have to extend it so that there is enough energy provided to launch the payload, and whether or not this could be repeated again. So, then we researched about at least flinging the payload into space and came upon SpinLaunch, a company that has now acquainted with NASA and is engineering a way to launch objects into orbit using a centrifuge machine, similar to how a slingshot would work where the object is flung into space at high velocities, so we used their contraption as out basis. Figure 2 below is the orbital launching system developed by SpinLaunch.

The location of where we put the point of launching our payload into space using the centrifuge machine by SpinLaunch would be an important aspect to investigate, especially the height at which it is at. Having the point of launch higher above sea level would reduce the amount of energy and force needed to fling the payload into orbit, meaning we could increase the mass of the payload compared to launching it below sea level. However, we couldn't have it at a place where it would be too difficult to launch such as Mt Everest as it would be too difficult to construct and maintain in its harsh conditions, so we decided to have the launch point be at the top of Mt Cook in NZ. This location would provide an extra boost to the centrifuge while also being practical and feasible.

Latitude will also have an impact on the tangential velocity of the Earth's surface. In other words, the rotational speed of the Earth will vary at different latitudes. This means that the location of the launch site will have an impact on the change in velocity for the payload to enter orbit. From this, we understand that launching at a lower altitude will mean that the weight of the payload will have to decrease.

The tangential speed formula can be given by:

$$v_t = \frac{2\pi \cdot r}{T} \cdot \cos(latitude)$$

v_t represents the tangential velocity of the Earth's surface. The radius of the Earth is represented by r. T is the sidereal rotational period in seconds, which is also known as the time for the Earth to rotate about its axis.

$$v_t = \frac{2\pi \cdot 6371000}{86400} \cdot \cos(0)$$

The equation above represents the tangential velocity at the equator. At the equator, the latitude is 0 degrees, which is represented by the cosine of 0. The 86400 represents the amount of seconds for the Earth to rotate about its axis once. The radius of the Earth has been converted into metres, which is approximately 6371000 m. Therefore, the equatorial tangential speed is approximately 463.3 metres per second.

Comparing this to Mount Cook, which has a latitude of 43.59° S.

$$v_t = \frac{2\pi \cdot 6371000}{86400} \cdot \cos(43.59)$$

This equates to approximately 335.57 metres per second. The approximate 130 metres per second difference in tangential speed means that a lower payload will be able to be carried at Mount Cook. Whereas if we launched at the equator, we'd be able to have a slightly heavier payload.

Figure 2

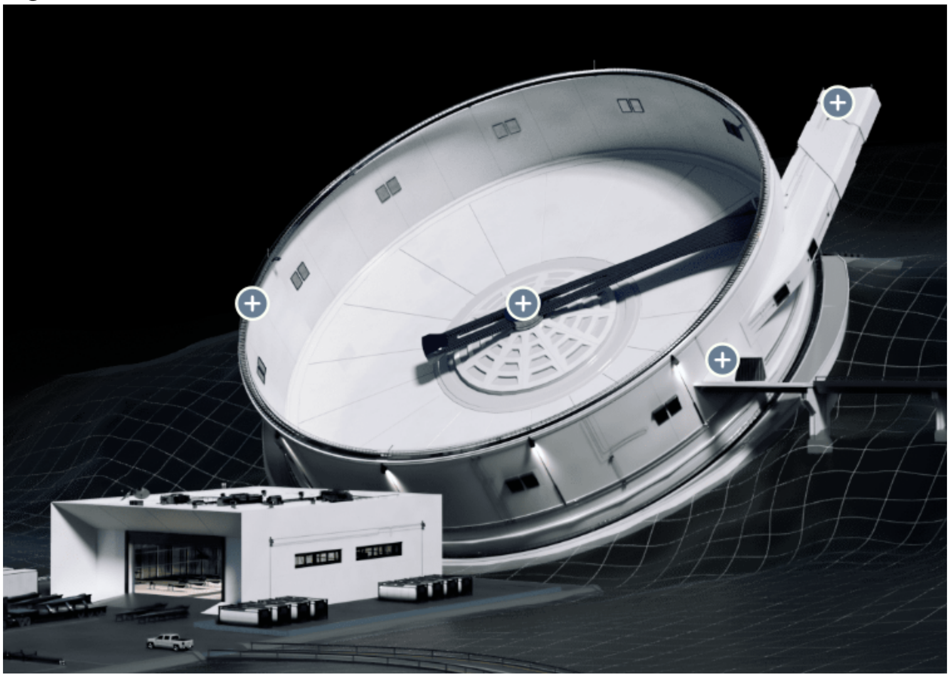


Image credit: SpinLaunch

Mathematical Modelling:

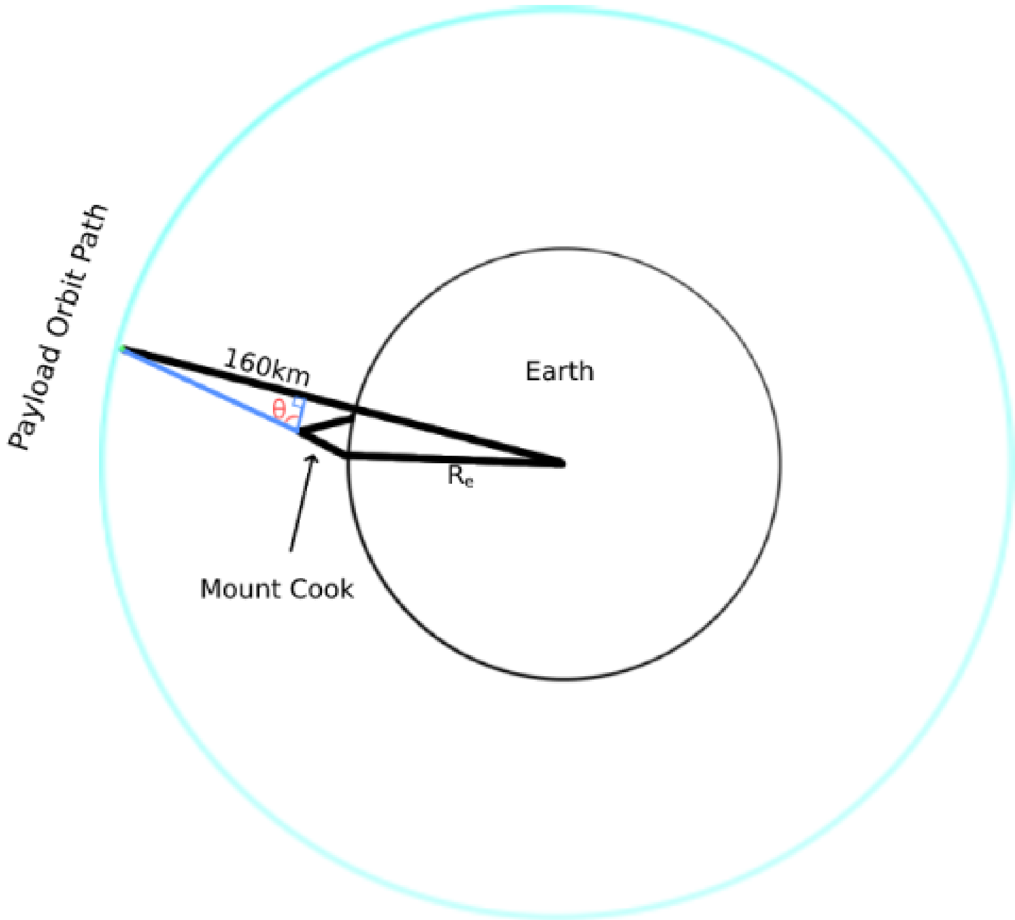


Diagram is not drawn to scale.
Low Earth orbit (LEO) is the closest orbit range that is relatively close to the surface of the Earth, objects in this orbit range have a period of 128 minutes or less to make a full rotation around the Earth (11.25 orbits a day). The LEO range is between 160 km - 2000 km in altitude. We chose 160 km as the altitude of our payload orbit path as that is the minimum distance from the surface of the Earth for an object to be considered as orbiting in LEO. R_e represents the radius of the Earth which is 6,371 km. We have decided that we would launch the payload from the top of Mount Cook (Aoraki), which stands 3,724 metres (3.724 km) above sea level. This would mean the payload would only have to travel 156.276 Km for it to be in low Earth orbit. We will be launching the payload at an angle of theta which we have decided to be 35 degrees. [3]
As for how to calculate the, maximum mass, the following calculations were performed:

$$F_{max} = \frac{m_p v_l^2}{r_m}$$

This is the maximum centripetal that the beam which rotates the projectile is capable of producing. In this equation, F_{max} is the maximum force, m_p is the mass of the payload, v_l is the velocity of the payload whilst it is rotating, and r_m is the length of the beam which is rotating the payload.

$$v_l = \sqrt{\frac{r_m F_{max}}{m_p}}$$

Solving for velocity

Since the Earth is huge, compared to the amount of motion the payload will do, we will assume that the motion is approximately akin to projectile motion.

When the vertical component diminishes due to gravity, the payload will enter orbital motion. From here we can find out the mass of the payload using the SUVAT equations.

$$v_f^2 = v_i^2 + 2ad$$

Where v_f is final velocity of the payload, v_i is the initial velocity, a is the gravitational acceleration (9.81 ms^{-2}), and d is the change in distance.

For this, we only need the vertical component of the velocity which is given by:

$$v_i \sin 35$$

(the angle of launch is 35° as per the SpinLaunch orbital launcher system)

$$= \sqrt{\frac{r_m F_{max}}{m_p}} \sin 35$$

Plugging this back into the SUVAT equation:

$$2(9.81)\left((160 - 3.7) \cdot 10^3\right) = \frac{r_m F_{max}}{m_p} (\sin 35)^2$$

Referencing back to the original SUVAT equation, v_f is 0 ms^{-1}, v_i^2 is $(r_m F_{max}/m_p)\sin^2 35$, a is -9.81 ms^{-2}, and d is $(160\text{-}3.7)(10^3)$, this is the displacement from Mount Cook to LEO.

In this scenario, we assumed F_{max} to be 22 million Newtons before breaking [3], though r_m is the actual value of the radius of the spinning arm, which is 45 metres.

Rearranging to find m_p:

$$m_p = \frac{(45)\left(22 \cdot 10^6\right) \sin^2 35}{2(9.81)\left((160 - 3.7) \cdot 10^3\right)}$$

This result gives 106.2 kg as a result of the maximum payload we can launch into orbit via a slingshot.

Conclusion:

Based on calculations of a mathematical model, we can expect that the maximum mass of the payload that we are able to “slingshot” into Lower Earth Orbit is 106 kg. This value was deduced by using the information we found about the SpinLaunch mechanism and how it used centripetal forces to send objects into orbit by spinning at very high velocities, and releasing it so that it is released at a tangential speed which is sufficient enough to get the payload into the orbital radius.

Discussion:

This result came about with a lot of assumptions mostly because of the limitations of current day mathematics and scarce research into non-combustion based methods of launching projectiles into orbit. One of our biggest hurdles came about when we had to compensate for the difference in strength of gravity as we went further up in altitude. This meant using lagrangian mechanics to try to formulate the way the Y coordinate of the velocity was changing with respect to time. Though when time came to solve the differential equation, the result was non-elementary. The difference was 5% between the strength of gravity at Mt Cook and LEO, but we decided to deem it negligible and carry on with our calculations using a constant gravitational acceleration of 9.81.

One of the most important conditions that should be met in this scenario is that $v_l\cos35=(GM_e/r_{orbit})^{0.5}$ where G is the gravitational constant, M_e is the mass of Earth and r_{orbit} is the radius of orbit from the centre of mass of Earth. This is because, $v_l\cos35$ is greater than the aforementioned value, then the payload will fly away from Earth, conversely if it is too little, it will fall back down to Earth.

Bibliography:

[1] Edwards, C., & Sun, T. (2022, April 15). Giant “space slingshot” set to launch NASA craft in 2022 revealed. New York Post. https://nypost.com/2022/04/15/giant-space-slingshot-set-to-launch-nasa-craft-in-2022-revealed/

[2] SpinLaunch to fling a NASA payload toward space. (n.d.). Freethink. Retrieved August 6, 2022, from https://www.freethink.com/space/spinlaunch

[3] Allain, R. (n.d.). Hurling Satellites Into Space Seems Crazy—but Might Just Work. Wired. Retrieved August 6, 2022, from https://www.wired.com/story/hurling-satellites-into-space/

NASA - How much of an advantage do equator-proximal launch sites provide? (n.d.). Space Exploration Stack Exchange. Retrieved August 6, 2022, from https://space.stackexchange.com/questions/998/how-much-of-an-advantage-do-equator-proximal-launch-sites-provide?noredirect=1&lq=1

Low Earth orbit. (n.d.). Www.esa.int. Retrieved August 6,2022, from https://www.esa.int/ESA_Multimedia/Images/2020/03/Low_Earth_orbit