Team 3009 Report 2012

Problem:

Next month Felix Baumgartner plans on breaking the world record for high altitude skydiving. He will make his jump from a capsule suspended beneath a balloon, at the edge of space. After Felix has landed, a remote triggering system will release the capsule from the balloon.

In the event that electronic tracking is unavailable, what size search area is required in order to retrieve the capsule?

Interpretation of Problem:

Felix Baumgartner is an Austrian skydiver and base jumper known for his high altitude and dangerous jumps. We understand that the problem proposed is in relation to his next jump, in which he will attempt to break the current world record of 102,800 feet (31333.4 m)[1] by approximately 17,200 feet (5242.56) and complete his jump from 120,000 feet (36576m)[2]. We understand that the capsule will also fall from this height, and that we are only required to calculate the area of the search, not the location in relation to the drop point.

Background Information on Drop

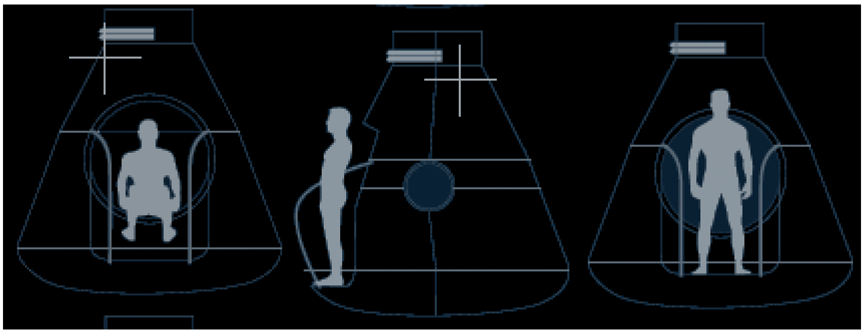
The jump is scheduled for the first two weeks of October 2012, and will launch from Roswell in New Mexico. A high altitude balloon which is approximately 600 feet high and filled with helium will lift Felix and his capsule to 120,000 feet for the drop.[3] After Felix jumps out of the capsule, the capsule will detach from the balloon and release a drogue parachute instantly to "stabilise and slow the capsules initial fall" [4] and bring it back to ground level.

Balloon and Capsule

"The balloon is made of polyethylene film that is only 0.0008 inches (0.002 centimetres) thick. The film is cut into banana peel–shaped sections called gores and heat-sealed together to form the balloon envelope. Although thinner than sandwich wrap, these strips in total cover 40 acres and weigh about 3,000 pounds. Vent ducts at the base of the balloon allow excess helium to escape. All this results in low weight and high strength and reliability, the ideal combination for a safe flight."[4]

The balloons can only withstand a maximum wind speed of 6mph (9.65606km/h), if wind speed exceeds this they are likely to postpone the launch.[5]

The capsule is used to ascend Felix to 120,000 feet and is similar to that used for space re-entry in shape. It measures 11 feet high and its base is 8 feet in diameter (see image below).[6]



[1]http://www.popularmechanics.com/outdoors/recreation/off-road/4273936
[2]http://www.redbullstratos.com/the-mission/what-is-the-mission/
[3]http://www.redbullstratos.com/technology/high-altitude-balloon/
[4]http://staging.site.redbullstratos.com/Science.aspx
[5]http://www.redbullstratos.com/science/meteorology/
[6]http://www.redbullstratos.com/technology/capsule/
http://www.redbullstratos.com/technology/parachute/

Factors which will effect search area:

The main factors which will influence the size of the search area will be descent time (of the capsule), the speed of the wind, and the direction of the wind. We are assuming that maximum wind speed is 9.65606km/h (see research for justification).[19]

Why we agree that Roswell in New Mexico is a suitable place:

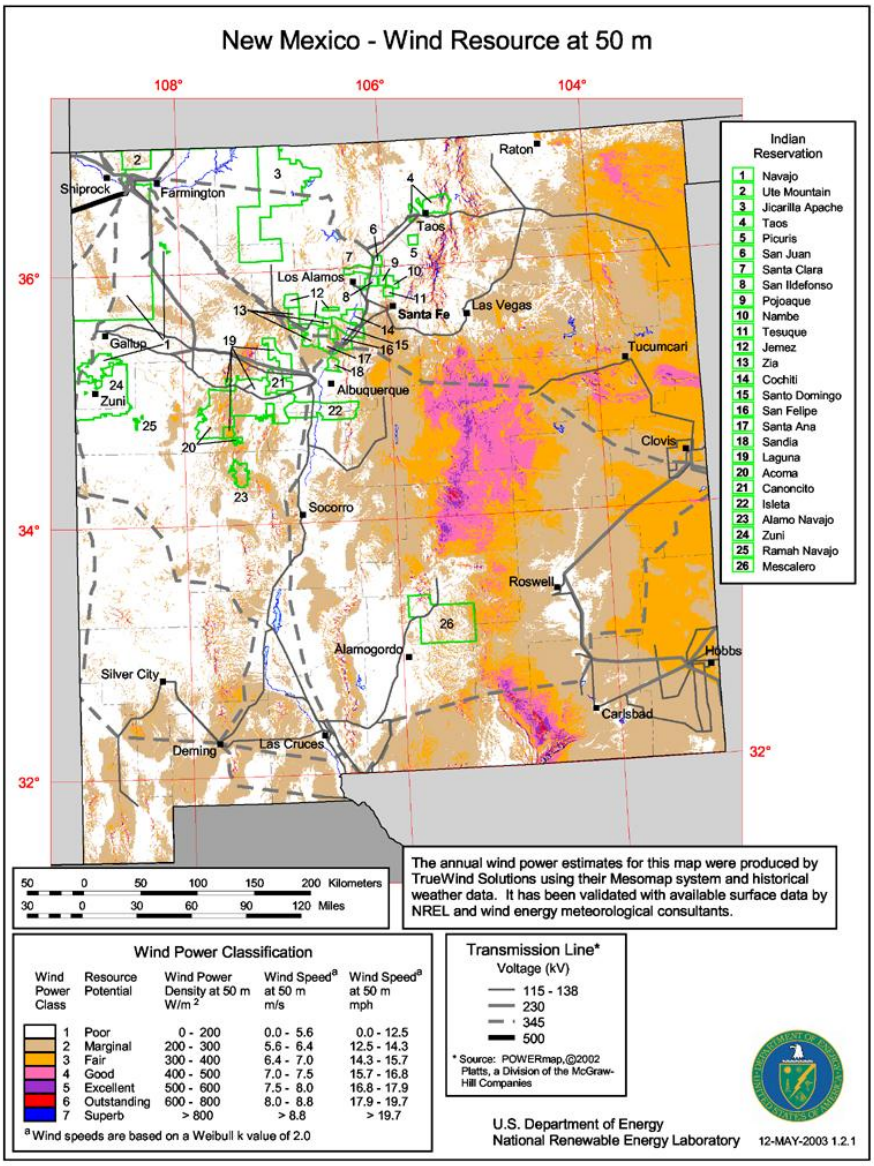
New Mexico is an ideal location for the capsule to land for a number of reasons including wind speed, sunny weather and large unpopulated areas. Figure 1, New Mexico weather charts, showing maximums for months[7]:

Figure 1

Year	Lowest Temperature (F)	Highest Temperature (F)	Warmest Minimum Temperature (F)	Coldest Maximum Temperature (F)	Average Minimum Temperature (F)	Average Maximum Temperature (F)	Mean Temperature (F)	Total Precipitation (In)	Total Snowfall (In)	Max 24hr Precipitation (In)	Max 24hr Snowfall (In)
2012	18	80	46	42	27.7	62.4	45.0	0.00	0.00	0.00	0.00
2011	6	75	36	36	20.2	59.1	39.6	0.00	0.00	0.00	0.00
2010	9	70	39	27	23.5	53.7	38.6	0.50	3.00	0.41	2.70
2009	13	78	36	39	23.0	61.6	42.3	0.00	0.00	0.00	0.00
2008	13	73	48	33	24.5	55.6	40.1	0.03	0.30	0.03	0.30
2007	14	74	42	31	24.2	47.8	36.0	0.86	7.60	0.50	5.50
2006	12	79	49	50	28.2	63.6	45.9	0.00	0.00	0.00	0.00
2005	21	73	47	44	31.6	58.4	45.0	0.62	0.00	0.28	0.00
2004	16	75	46	38	29.6	57.5	43.6	0.00	0.00	0.00	0.00
2003	18	75	37	40	27.6	60.2	43.9	0.05	0.00	0.04	0.00
2002	18	78	46	30	28.0	58.4	43.2	0.52	2.20	0.45	2.20

The above weather chart shows that New Mexico has very little maximum rainfall or snowfall making it ideal for the launch, drop and land site. Along with very little snow and rainfall, wind is also an important factor; background information on the balloon for justification on wind speeds. Figure 2 on the next page shows the average wind speeds for areas in New Mexico, and shows that Roswell has very little and is an ideal area.[8]

Figure 2



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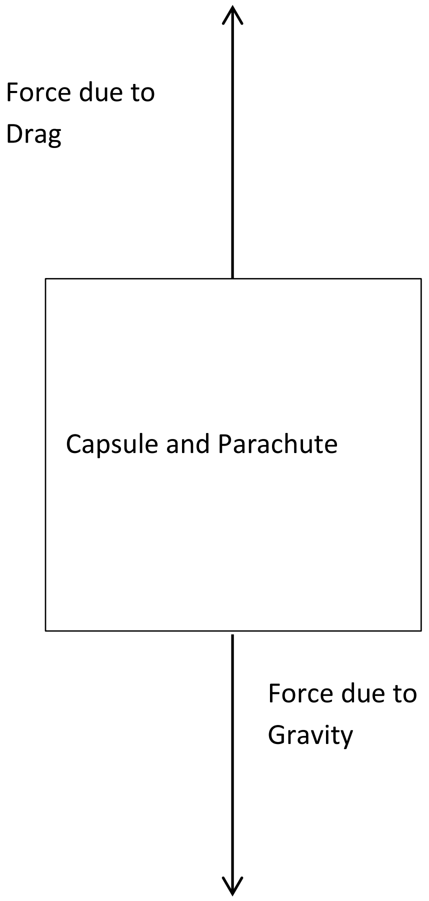
7 http://weatherwarehouse.com/WeatherHistory/PastWeatherData_RoswellIndustrialArptPark_Roswell_NM_January.html

8 http://www.nmsu.edu/~tdi/Wind/nm_50m_800.jpg

Time taken for descent:

When the capsule travels through the air it will be under the influence of two distinct vertical forces: the force due to gravity and the drag force. The force due to gravity will act in the downwards direction while the drag force will act in the upwards direction. This can be seen in the diagram to the left the gravity force will, in reality, change as the capsule gets closer to the earth. However, because the relative height of the capsule is very tiny compared to the size of the actual planet we assumed in our calculations that the size of the gravitational acceleration induced in the capsule and its parachute remains constant at a value of 9.81 ms^{-2}. Therefore the force due to gravity can easily be calculated because the acceleration and mass are constant and:



$$F = m \times a$$

$$F = 1315 \times 9.81$$

However, the opposite force - drag - is more of a challenge. Drag can be several different types, namely pressure drag[21], caused by differences in pressure as the air is split around the parachute. This is dependent on the shape of the parachute. The parachute used for the launch is a drogue parachute which has a hemispherical shape. Therefore, the air will accumulate inside the parachute creating a high pressure and high pressure drag. The second form of drag that is significant is called skin friction drag[22]. This is caused by the actual friction between the air molecules and the surface of the object which is in motion. This acts tangentially to the surface of the friction. In this case the surface is a parachute which has a large portion of the force tangentially horizontal and therefore not considered part of the total drag force. The overall force caused by the drag on the parachute is given by the following equation[23]:

$$F_D = C_d \times A \times v^2 \times p \times 0.5$$

In which A = area of shape

Cd = the drag constant of the shape

V = the velocity of the parachute at that instantaneous moment

p = the density of air at that instantaneous height above the ground

Fd = the force due to drag on the object

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[21] http://www.griffin-helicopters.co.uk/note/TypesOfDrag.htm
[22] http://www.griffin-helicopters.co.uk/note/TypesOfDrag.htm
[23] http://www.grc.nasa.gov/WWW/k-12/airplane/drag1.html

The Area of the Shape

In this equation, A is the area of the object which is used to determine its approximate shape. When the parachute descends the actual capsule, and therefore the mass of the overall shape, will be suspended below the centre of the parachute. Therefore, the parachute will face the direction of the capsule's motion during the entire flight. Assume here that there are no changes in parachute shape due to wind effects/air etc. The area used in the equation can vary: either using the entire surface area of the shape in order to show the overall skin friction drag or by using the area of the shape perpendicular to the direction of the motion of the object. This would account for the pressure drag being the larger of the two forces. In a parachute travelling at a relatively large velocity, the drag created by the pressure displaced is generally going to be larger than the drag created by the skin friction.[24] This is given by a large Reynolds's number, which is the ratio of the pressure drag in comparison with the skin friction drag. In the case of the parachute, the area used is the second option, given by

$$A = \pi \times r^2$$

The area of a 2D representation of a hemispherical parachute is given by a circle with the equation above for the area of the circle. In that above equation, r is the radius of the circle. The radius of the parachute required is assumed to be 3.07 meters in order to support the mass of the capsule at a reasonable final velocity when it hits the ground in New Mexico[25] of 50 feet per second.

The Drag Co-efficient

Cd is the drag co-efficient of the particular object being tested. The drag co-efficient is generally found experimentally. As finding the perfect sum of the different components of drag is very difficult, the drag co-efficient of the particular object is generally found experimentally using a wind tunnel[26] to measure drag and dividing out to find the co-efficient. Therefore, for the purpose of this report the existing drag co-efficient of a standard drogue parachute was used. This is a value of 1.5. an important assumption in the use of the parachute to find both area and drag co-efficient is that it is assumed that by comparison to the very large drag generated by the parachute, the drag generated by the actual capsule is negligible, as it is a far more aerodynamic object. Most of the drag generated by the capsule would end up reducing the drag produced by the parachute in any case, so the assumption is sensible.

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24 http://www.grc.nasa.gov/WWW/k-12/airplane/drag1.html
25 http://www.rocketryplanet.com/info-central.org/recovery_drogue.shtml
26 http://www.grc.nasa.gov/WWW/k-12/airplane/drag1.htm

Air Density at different altitudes:

Air density is a function of temperature and pressure shown by the equation below. [9]

$$\rho = \frac{p}{R_{\text{specific}} T}$$

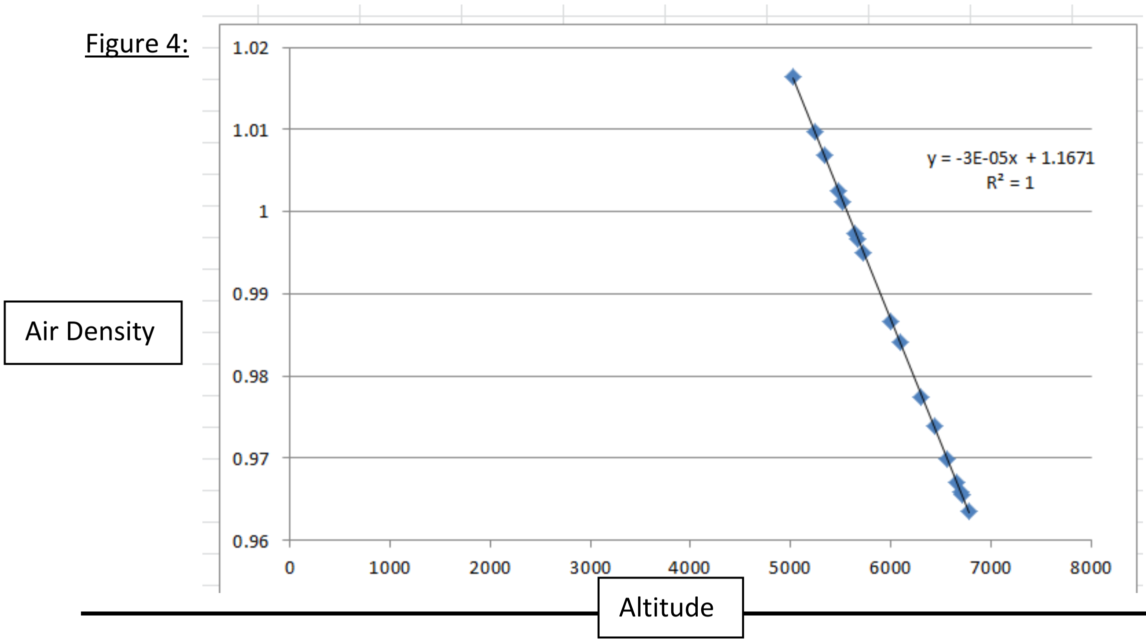
As we know altitudes, but not air pressure at different altitudes it is impossible to work out air density. Instead of calculating, we found values for different altitudes shown below in figure 3[10]:

Figure 3

Time	Temp	Humidity	Barometer	Dew Point	Grains	Air Density	Density Altitude
09/21 11:00 PM	70	25%	25.89 Hg	33	31.3	84.30 %	5732 ft.
09/22 02:00 AM	66	29%	25.89 Hg	33	31.6	84.93 %	5485 ft.
09/22 05:00 AM	62	35%	25.89 Hg	34	33.2	85.55 %	5245 ft.
09/22 08:00 AM	69	28%	25.89 Hg	35	33.9	84.41 %	5689 ft.
09/22 11:00 AM	82	19%	25.89 Hg	36	35.5	82.35 %	6498 ft.
09/22 02:00 PM	88	14%	25.89 Hg	33	31.7	81.52 %	6829 ft.
09/22 05:00 PM	86	14%	25.89 Hg	31	29.7	81.86 %	6695 ft.
09/22 08:00 PM	74	23%	25.89 Hg	34	33.0	83.64 %	5992 ft.
09/22 11:00 PM	68	31%	25.89 Hg	36	36.3	84.52 %	5645 ft.
09/23 02:00 AM	64	37%	25.89 Hg	37	37.7	85.14 %	5405 ft.
09/23 05:00 AM	59	44%	25.89 Hg	37	37.6	85.96 %	5087 ft.
09/23 08:00 AM	66	33%	25.89 Hg	36	36.0	84.85 %	5518 ft.
09/23 11:00 AM	80	20%	25.89 Hg	35	35.0	82.67 %	6373 ft.
09/23 02:00 PM	86	15%	25.89 Hg	33	31.8	81.82 %	6711 ft.
09/23 05:00 PM	85	14%	25.89 Hg	31	28.8	82.03 %	6628 ft.
09/23 08:00 PM	73	20%	25.89 Hg	30	27.7	83.89 %	5890 ft.
09/23 11:00 PM	68	26%	25.89 Hg	32	30.4	84.64 %	5600 ft.

The figures are graphed below on figure 4:

Figure 4:



[9]http://en.wikipedia.org/wiki/Density_of_air

[10]http://www.airdensityonline.com/trackForecast.php?trackname=Southern%20New%20Mexico%20Speedway&bar=25.892653286489

From this graph (figure 4), we can use a trend line to calculate air density at different altitudes. Although the altitudes for the given data are similar, we can assume that the relationship is linear as $R^2 = 1$.

The Velocity of the Object

The velocity of the parachute is simply how fast it is going at any point in time. If the parachute is moving faster more air will be parted per second, thus there will be more drag

When the capsule and its parachute fall together, they are initially falling out of an area in the stratosphere at 36 576 metres above sea level. As explained above the density of air is a function of height, with a large height producing a very low air density. Therefore as the capsule falls at this height it contacts a very small number of air molecules and accelerates quickly because the force due to drag is small and the force due to gravity is, by comparison, very large. It accelerates like this until it reaches a terminal velocity at which point the drag force is equal and opposite to the force due to gravity, so there would be no net forces on the capsule. Thus:

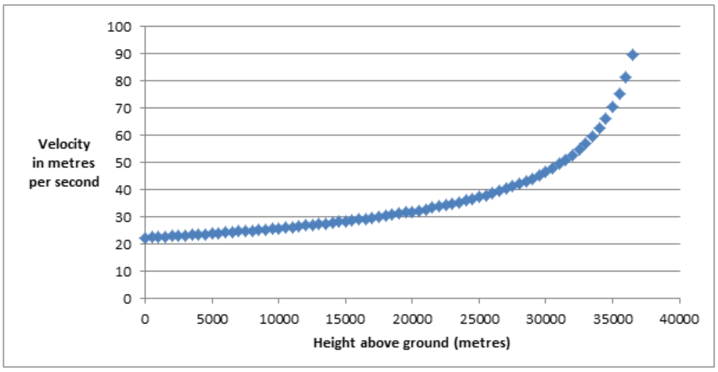
$$F_g = F_D$$

$$\text{Therefore } m \times a = Cd \times A \times v^2 \times 0.5 \times \rho$$

Because at this point the two forces are equal, the equations can be rearranged to give $v = \sqrt{\frac{2mg}{\rho C_d A}}$

The initial acceleration of the object is very fast so the time taken for this first part of the fall is very small in comparison to the time taken for the entire journey. Therefore, as d = v x t, a small time will result in a small distance being covered during this part of the flight. Because this time is very small, it can be assumed to be negligible in comparison to the overall time taken for the entire fall of the capsule. Therefore the model used for the second half of the fall can be used to estimate the time taken for the entire fall.

However, as the object falls the height decreases. Therefore the air density, which increases in a linear way as height decreases. Because the air density (ρ) increases the force due to drag also increases as F_D is proportional to air density. Therefore at the same velocity the drag force is greater than the force on the object due to gravity, so the object accelerates upwards as the forces are not balanced. Therefore the velocity of the entire capsule decreases as the air density increases. Because velocity is proportional to the inverse of the square root of the air density, the decrease of velocity caused in this way is not linear, but velocity does decrease as the capsule descends, so the initial terminal velocity of the capsule is also the largest velocity that the capsule achieves during its flight. In fact, the change is given in the graph to the right:



The time taken is important as it shows how much time the wind would have to act upon the capsule during its flight. The velocities calculated in this section do not include the horizontal components of velocity which are calculated in a subsequent section. The sole purpose of this is to find the time taken for the fall, which can be used to find how long there is for the horizontal force due to the wind to act upon the capsule.

To work out the terminal velocity at any point during the drop, we used the equation $v = \sqrt{\frac{2mg}{\rho C_d A}}$

Where,

- m is the mass of the capsule (1315kg)
- g is the acceleration of gravity (-9.8ms^{-2})
- p is the air density kgm^{-3} (-0.00003h + 1.167)
- Cd is the drag coefficient (1.5)
- A is the perpendicular area to the angle of motion, of the drogue parachute (29.7m^2)

The air density is given as shown above, by the equation $\rho = -0.00003h + 1.167$

$$\therefore \quad v = \sqrt{\frac{2 \times 1315 \times -9.8}{(-0.00003h + 1.167) \times 1.5 \times 29.7}}$$

$$v = \sqrt{\frac{-578.5}{-0.00003h + 1.167}}$$

Then we were able to use this equation to plot points for speeds every 500 metres and find the mean velocity for the drop.

height	velocity
36500	89.63661
36000	81.54401
35500	75.30982
35000	70.31674
34500	66.20103
34000	62.73256
33500	59.75774
33000	57.16959
32500	54.89099
32000	52.86479
31500	51.04758
31000	49.40575
30500	47.91278
30000	46.54747
29500	45.29259
29000	44.13402
28500	43.06004
28000	42.06083
27500	41.1281
27000	40.25479
26500	39.43485
26000	38.66306
25500	37.93487
25000	37.24634
24500	36.59399
24000	35.97476
23500	35.38594
23000	34.82511
22500	34.29013
22000	33.77907
21500	33.2902
21000	32.82196
20500	32.37294
20000	31.94186
19500	31.52755
19000	31.12895
18500	30.7451
18000	30.37511
17500	30.01817
17000	29.67352
16500	29.34047
16000	29.01839
15500	28.70669
15000	28.40482
14500	28.11228
14000	27.8286
13500	27.55334
13000	27.28608
12500	27.02646
12000	26.7741
11500	26.52869
11000	26.2899
10500	26.05745
10000	25.83105
9500	25.61046
9000	25.39542
8500	25.18571
8000	24.98111
7500	24.78142
7000	24.58644
6500	24.396
6000	24.20991
5500	24.02801
5000	23.85015
4500	23.67619
4000	23.50598
3500	23.33939
3000	23.17629
2500	23.01656
2000	22.86009
1500	22.70676
1000	22.55649
500	22.40915
0	22.26467
mean velocity	35.95386

And then taking the mean velocity from the drop height 36580m to the landing height 0m, we worked out that the mean velocity for the duration of the drop to be 35.95ms^{-1} and then using $v = d/t$ we can use the drop height of 36580m and a mean velocity of 35.95 the time taken must be 1017 seconds from the time the capsule is dropped to the time it lands.

Calculating the maximum wind speed:

All of this assumes that the wind direction is Horizontal

A rise in altitude translates to a rise in wind speed. This is because winds closer to the surface of the earth face more obstacles that cause winds to move in different directions and thus cancel each other out[11].

The balloon that lifts the capsule can only be launched at wind speeds of 2.68m/s[5.]

Using the wind profile power law where

"The wind profile power law relationship is:

$$u/u_r = (z/z_r)^\alpha$$

Where' u' is the wind speed (in meters per second) at height z (in meters), and u_r is the known wind speed at a reference height z_r. The exponent (α) is an empirically derived coefficient that varies dependent upon the stability of the atmosphere" [12]

Alpha is dependent on whether the conditions are stable or unstable, in order for a balloon to rise the air has to be unstable as during periods on unstability[13]

If air is unstable vertical motions is favored, as the balloon is warmer than its surrounding air, as a result the balloon rises. The balloon for this jump has to rise to 36576m therefore the conditions have to be unstable.[14]

This allows us to use the wind profile law, the regions around Roswell is sparsely inhabited therefore the Hellman's constant is 0.27.[15]

Speed at highest altitude = Max speed for balloon launch multiplied by (Max height/10)^0.27.

The speed at height 10m above ground is assumed to be the same as speed of wind at point of balloon launch.

=2.68 times (3657.60^0.27)

=24.55m/s (Assuming wind speeds are the same throughout the day) this value is therefore one of the possible values of the wind speed the capsule faces as it descends.

[11]http://en.wikipedia.org/wiki/Wind_gradient
[12]http://en.wikipedia.org/wiki/Wind_profile_power_law
[13]http://www.sundanceballoons.com/frequently-asked-questions/
[14]http://www.aviationweather.ws/034_Stability_and_Instability.php
[15] http://en.wikipedia.org/wiki/Wind_gradient

According to [16] and [17] and [18] average wind speeds near the ground will be between 3-5m/s

This does not mean that the wind speeds at different times of the day will not be greater or lesser.

According to [19] the maximum possible wind speed in the first 2 weeks of October is between 6.4 to 6.8m/s and the minimum possible wind speed is 0.1 or nearly 0m/s second at any point.

Using the wind profile law this gives us the extremities of the possible wind speeds at 36756m

At ground speed 6.8m/s the max speed is 62.31m/s at 36576m.

At ground speed 0.1m/s the max speed is 0.91m/s.

Wind Direction

According to these weather websites (references 16,17,18) the wind direction during the first 2 weeks of October is most of the time from the south.

Average % time from the North is 15%.
Average% time from North East is 6 %
Average % time from East is 5%
Average %time from South East is 16%
Average % time from South is 22%
Average % time from South West is 8%
Average % time from West is 8.6666%
Average % time from North West is 8.33333%

The rest of the time the wind speed is zero according to the websites referenced.

This fits with the data on wind direction from reference 20.

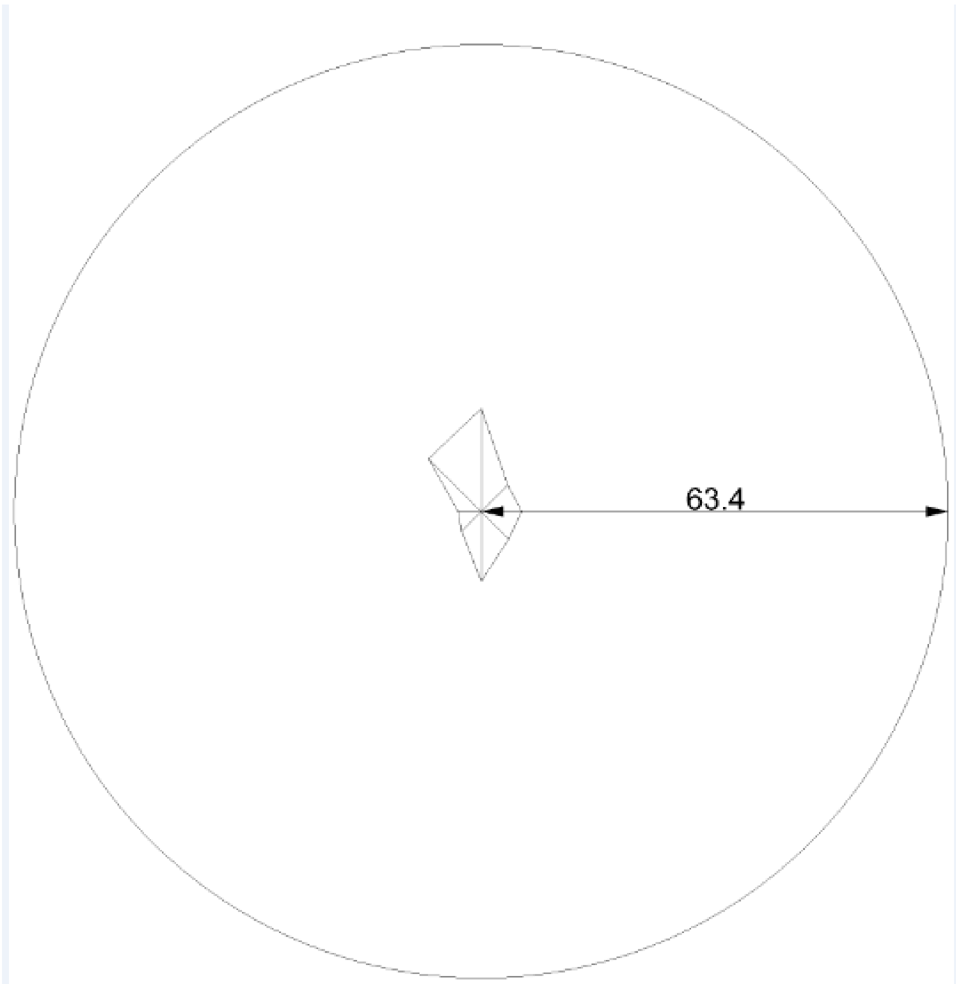
[16] http://weatherspark.com/averages/31497/10/7/Roswell-New-Mexico-United-States
[17] http://weatherspark.com/averages/31497/10/14/Roswell-New-Mexico-United-States
[18] http://weatherspark.com/averages/31497/10/1/Roswell-New-Mexico-United-States
[19] http://www.redbullstratos.com/technology/parachute/
[20] http://weatherspark.com/averages/31497/Roswell-New-Mexico-United-States

Area of Capsule Landing

As calculated above, the time in air is 1017 seconds. We are using the wind speed as the speed of the object as this is the maximum possible velocity it can travel in the horizontal component. The safe speed for the balloon launch at ground level is 2.68m/s but as wind speeds can change during the day and thus after the balloon launch the ground speed can change, the windspeed we used is the maximum possible groundspeed at Roswell during that time of the year, this is 6.8m/s and as calculated above translates to 62.31m/s at 36576m.Although the speed acting on the capsule falls as its altitude decreases while it descends, using the maximum speed at the top in order to calculate distance means that the distances calculated will be the maximum.

As speed is distance divided by time and the time was calculated above (1017s) the distance away from the launch point is 63.369Km

Figure 5



Using Area of a circle formula, area= πr^2

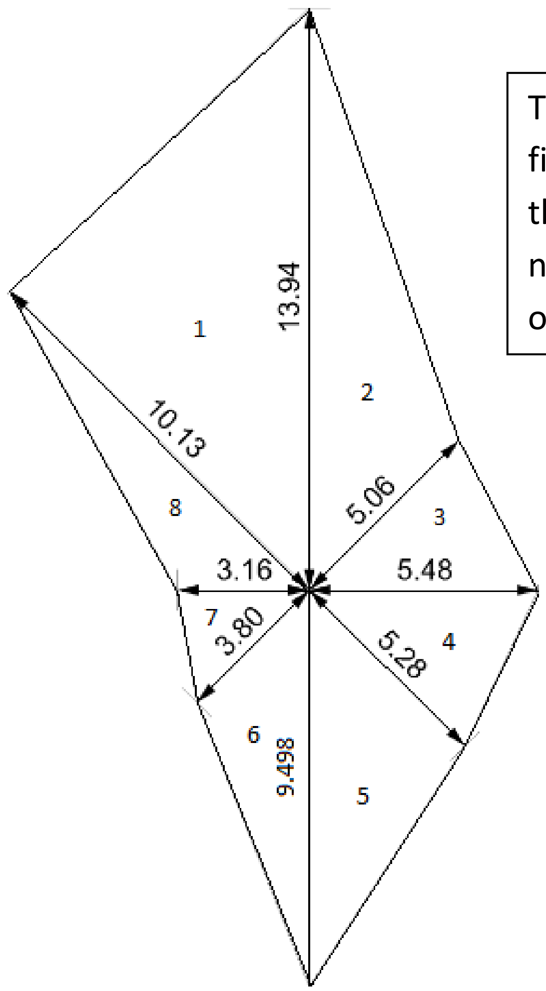
The total area of the circle is 12615Km^2

The area above however assumes that the wind direction is constant throughout the 17 min drop, this may not, be so and assuming the capsule is acted upon by winds from all directions in similar proportions as has always been in Roswell, New Mexico, we calculated different speeds from different wind directions and thus different distances for each wind velocity and direction.

The time used was 1017 seconds in order to calculate distance, as the speed max(62.31m/s) had already been divided into proportions based on the values under heading "Wind Direction"

Average speed is 9.34m/s from the North
Average speed is 3.73m/s form the South East
Average speed is 3.11m/s from East
Average speed is 9.96m/s form South East
Average speed is 13.941 from South
Average speed is 4.98m/s from South West
Average speed is 5.39m/s form west
Average speed from North west is 5.19m/s
Thus using Speed =Distance over time formula the distances were calculated, resulting in a Diagram like this.

Figure 6



The central point on both figure 5 and 6 may not be the central point under no wind, due to the spin of the earth.

Area Calculations (likely landing):

Using 0.5absinC, we calculated the areas of each triangle and added them up to find total area.

Area of a Triangle:

A = 0.5 x a x b x sinC

C is 45 degrees for all triangles.

Sin45 = 0.7071

Area 1:

A = 0.5 x 10.13 x 13.94 x 0.7071
A = 49.92km^2

Area 2:

A = 0.5 x 10.13 x 5.06 x 0.7071
A = 18.12km^2

Area 3:

A = 0.5 x 5.48 x 5.06 x 0.7071
A = 9.80km^2

Area 4:

A = 0.5 x 5.48 x 5.28 x 0.7071
A = 10.23km^2

Area 5:

A = 0.5 x 5.28 x 9.49 x 0.7071
A = 17.73km^2

Area 6:

A = 0.5 x 3.80 x 9.49 x 0.7071
A = 12.76km^2

Area 7:

A = 0.5 x 3.80 x 3.16 x 0.7071
A = 4.25km^2

Area 8:

A = 0.5 x 10.13 x 3.16 x 0.7071
A = 13.61km^2

Total Area for Likely Landing Spot

49.92 + 18.12 + 9.80 + 10.23 + 17.73 + 12.76 + 4.25 + 13.61 = **136.42km^2**

Conclusion:

The search area needed which covers the maximum displacement of the point of release of the capsule can be calculated by the maximum wind horizontal speed, multiplied by the time taken for the object to fall the vertical distance. This time can be found by finding the maximum velocity of the capsule at each different air density.

We thus have 2 possible areas that a search team will have to cover,

The search area that will guarantee success in finding the capsule will be the one that uses max speed and assumes speed is in the same direction throughout the fall .This is the circular diagram which has an area of 12615 Km^2,

The second area is much smaller and takes into account all wind speeds acting on the object in the proportions of wind directions around Roswell.

This resulted in a total area of 136.42Km^2

As this is a much smaller area it will make sense for the team to look in this area first as seen from the diagram (If they know the point at which the capsule started to descend relative to launch point.).

The search area max however will only be 12615Km^2.