# Team 1064

"What is the fastest humanly possible time for the Olympic 100m freestyle event?"

The 100m freestyle long course Olympic world record currently stands at 47.05 seconds, set by Eamon Sullivan in the 2008 Beijing Olympics.[1] 100 years earlier, the record was 65.6 seconds,[2] nearly 19 seconds slower. In the future, how fast could a human theoretically push themselves to lower these records even further?

Our final time, using an approach of finding the maximum speed at which a swimmer could travel in each section of the race was **34.8 seconds**, about 12 seconds faster than the current world record.

## Approaching the Problem

### Fastest humanly possible time

We have defined the fastest humanly possibly time as the shortest time someone who is considered human could complete the course without assistance from another.

The physiological differences between male and female athletes are significant, to the extent that the advantages they allow males to attain, for example greater oxygen consumption, outweighing the tighter swimwear regulations[3]. As such it is assumed a male athlete will be capable of setting the fastest possible time. It is also assumed the swimmer's stroke will be a front crawl, as this has proven to be the most effective in achieving freestyle records[4].

### Olympic 100m freestyle

The Olympic 100m freestyle uses a "long course" (50m length) pool, with a depth of between 2m and 3m.[5] Swimmers dive in from a platform above the water's surface. Swimmers are allowed underwater in the first 15m of each length, after which they must resurface. [6] Swimmers must wear swimwear which meets the regulations set by FINA[7] as of 2009. These rules are strict on materials and sizes[8], and are intended to remove the substantial advantage of very buoyant and streamlined swimwear.

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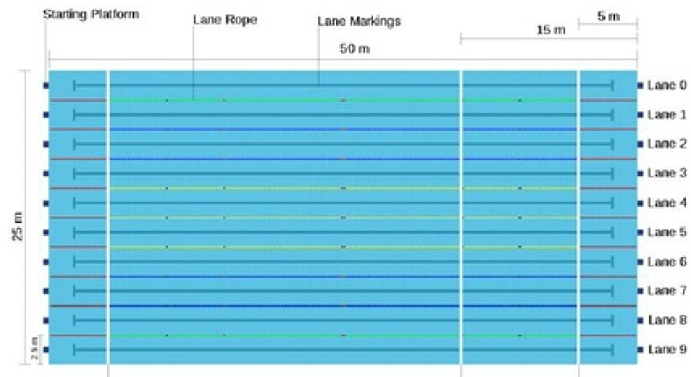
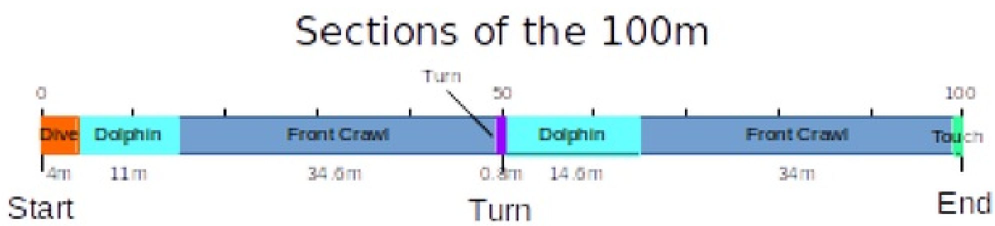
[1] https://en.wikipedia.org/wiki/World_record_progression_100_metres_freestyle
[2] "The Olympic Games – The 100 metres race". The Times. London: Alfred Harmsworth. 21 July 1908. p. 13.
[3] http://work.chron.com/physiological-differences-between-male-female-athletes-20627.html
[4] https://books.google.co.uk/books?id=cSSW4RhZOiwC&pg=PA219&dq=breaststroke&hl=en&sa=X&ei=215ST8raAsK48gPyjL3wEQ#v=onepage&q=front%20crawl&f=false
[5] https://www.fina.org/sites/default/files/finafacilities_rules.pdf
[6] https://www.kiefer.com/blog/15-meter-resurfacing-marker-underwater-swimming-rule
[7] http://www.fina.org/
[8] http://www.fina.org/H2O/docs/rules/FRSA.pdf

## Distances to Travel



[9]

We have segregated the 100m swim into six components: the flight portion of the dive, the underwater portion of the dive (remainder of the first 15m), the next 34.6m of front crawl, the turn (0.8m total), 14.6m underwater dolphin kick post the turn and the final 34m of front crawl. The swimmer's arm extends about 1m, meaning we can ignore this part at the end.




# Basic Mathematical Model

Our basic mathematical model makes the assumption that to maintain a constant velocity (which we believe a well-trained athlete will do during the Olympic 100m freestyle), an athlete will need to produce a constant power output. As they move through the water, all energy they expend will be used to maintain this constant velocity as work against the drag force.

Our approach is to determine the predicted maximum velocity of each main portion of the swim, and use this to calculate the overall minimum time of the 100m. We will assume our optimum human is operating at maximum power output for each portion and using all of this power to work against the drag force. As a result, we can calculate a predicted velocity for each portion of our optimum human's swim.

We are considering three different types of drag: form drag, lift drag, and wave drag. We will determine the relationship between power output and velocity for each type of drag, and thus we will determine the overall relationship between power output and velocity.

For all three types of drag, we can use the following equation to convert drag force equations into power equations:

[9] http://www.sciencekids.co.nz/images/pictures/sports/olympicswimmingpooldiagram.jpg

$$P = Fv$$

This assumes, of course, that the velocity with which our swimmer moves is constant for each section of his swim, and that the force the swimmer is working against remains constant. In the context of Olympic swimmers, we believe this to be a reasonable assumption.

# Components of Mathematical Model

## Frictional Drag

Frictional drag is proportional to velocity of the swimmer, in contrast to form and lift drag which are proportional to velocity squared, and wave drag which is proportional to velocity cubed. Therefore at high speeds ( greater than 1.5 metres per second) frictional drag contributes relatively small percentage of the total drag and thereby we deemed it to have negligible effect (frictional drag is also relatively complicated to calculate) and didn't include it as part of the overall calculation of drag[10]. At the speed of 2 meters per second frictional drag contributes 3% of the overall drag force[11], supporting our decision to not include frictional drag as part of our overall drag calculation.

## Form Drag

Form drag is the drag as a result of an area of an object moving through a fluid.

$$F_D = \tfrac{1}{2}\rho v^2 C_D A$$

$F_D$ is the drag force, which is, by definition, the force component in the direction of the flow velocity,

$\rho$ is the mass density of the fluid,

v is the flow velocity relative to the object,

A is the reference area, and

$C_D$ is the drag coefficient – a dimensionless coefficient related to the object's geometry and taking into account both skin friction and form drag.

Thus the component for our mathematical model concerning form drag is:

$$P = \tfrac{1}{2}\rho v^3 C_D A$$

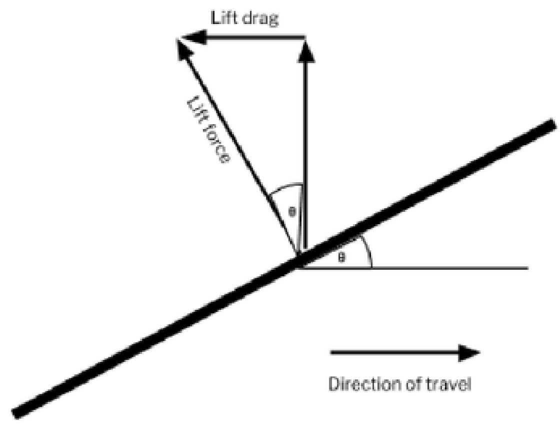
## Lift Drag

Swimmers, regardless of how skilled or well-trained, often end up swimming at a slight incline. As a result, the lift force gains a horizontal component pushing against the direction of the swimmer's movement - lift drag.

Regular lift force is given by:

[10] http://coachsci.sdsu.edu/swim/bullets/forces4.htm
[11] http://www.fade.up.pt/docentes/leandromachado/biomecanica/Hydrod-Drag.pdf

$$F_{lift} = \frac{1}{2}\rho v^2 C_L A$$




The horizontal component causing lift drag is thus:

$$F_L = \frac{1}{2}\rho v^2 C_L A \sin\theta$$

$F_L$ is the lift drag force

$\theta$ is the angle of incline and

$C_L$ is the lift coefficient

All other variables are the same as above.

The component for our mathematical model concerning lift drag is thus:

$$P = \frac{1}{2}\rho v^3 C_L A \sin\theta$$

## Wave Drag

Wave drag, in the case of the front crawl portion of the swim, forms a major part of the total drag. Wave drag is caused by a loss of energy is used to produce waves on the surface of the water. However, unlike the other forms of drag, relatively little is known about wave drag and there are no sources giving a reliable equation for it. One source suggests that wave drag increases with the cube of velocity[12], so we will assume this to be thus. As we do not have the specific parameters for this, we will use an arbitrary scaling constant for this cubic.

$$F_D = kv^3$$

$F_D$ is the wave drag

k is the wave drag constant and

v is the velocity

The component of our mathematical model concerning wave drag is thus:

[12] http://coachsci.sdsu.edu/swim/bullets/forces4.htm

$$P = kv^4$$

## Overall Equation

For the underwater (dolphin kick) portions of the swim, wave drag is not a factor, and $\theta$ is assumed to be 0 (only form drag needs to be taken into account), thus the total power is:

$$P = \frac{1}{2}\rho v^3 C_D A$$

For the freestyle portions of the swim, wave drag must be included in the calculation.

$$P = \frac{1}{2}\rho v^3 A\ (C_L sin\ \theta + C_D)\ +\ kv^4$$

# Defining Our Swimmer

The intention here is to define the ideal swimmer still in the realm of what can be considered human. The tallest person who ever swam competitively was 2.1m, so we will take this as the height of our ideal swimmer (this appears to be approximately the point where the maximum benefits of height can be reaped whilst avoiding the negative effects of pituitary gland problems).

We will assume his shoulder width is fairly wide (0.6m is fairly high according to this discussion[13]), which is the usual sort of build for swimmers.

We also assume he is perfectly skilled at swimming, thereby utilising his power output 100% efficiently. Furthermore, he minimises time spent in the front crawl portions, and maximises time spent in the underwater dolphin kick portions, thereby maximising his overall speed (as the dolphin or fish kick portion is faster than the front crawl).[14] The dolphin kick is limited to only the first 15m of each length, and we are assuming that he spends all 15m in this phase (aside from time he spends diving and turning).

# Constants Defined

## Density of Water

$\rho$ = 997 $kgm^{-3}$

This is the mass density of water at the regulation temperature of 25-28 degrees celsius.[15] [16] This the relevant fluid exerting a drag force.

## Drag Coefficient

$C_D$ = 0.4

[13] http://forum.bodybuilding.com/showthread.php?t=136723841
[14] http://nautil.us/issue/25/water/is-this-new-swim-stroke-the-fastest-yet
[15] https://www.fina.org/sites/default/files/finafacilities_rules.pdf
[16] http://water.usgs.gov/edu/density.html

This is slightly less than values measured at high speeds for swimmers in an experiment,[17] we believe that our swimmer can reduce his area and perfect his technique.

## Lift Coefficient

$C_L = 0.2$

There is a lesser effect of lift compared to drag for propulsion and drag,[18] hence approximately half as large a coefficient.

## Surface Area Pushing Against Water

$A = 0.21\ m^2$

We will model this as a rectangle, with the shoulders and the body depth as the length of the sides. Taking 0.6m as our swimmer's shoulder width, and assuming his proportions are roughly the same as the average person[19], our swimmer's body depth will be around 0.36m. This gives a total area of approximately $0.21m^2$.

## Angular Displacement from Parallel

$\theta$ = assume optimal 0 degrees during underwater dolphin kick phase; about 7 degrees against the horizontal water surface during front crawl phase

To allow a 2.1m tall individual 40 cm of space for kicking, 7 degrees would be required. The angle would be given by $\tan\theta = 0.4/2.1$ which gives $\theta = 10.8\ degrees$. However, since most of the swimmer's body will be flat against the horizontal water surface (until below the waist), as opposed to the straight line angle, 7 degrees would be more appropriate.

## Maximum Human Power Output in Swimming

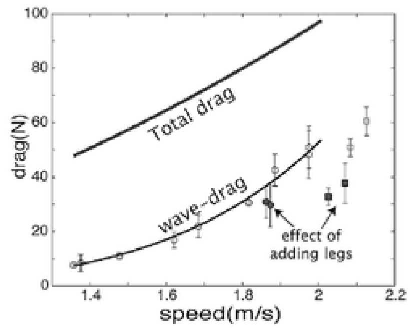
P = 1200 W

The maximum human power output for a short period of high-intensity exercise such as swimming has been recorded at 1200W[20].

## Wave Drag Coefficient

k (wave drag coefficient) = $4.2\ kgsm^{-2}$

[17] http://www.fade.up.pt/docentes/leandromachado/biomecanica/Hydrod-Drag.pdf Page 37
[18] http://www.sportsci.org/news/biomech/skeptic.html
[19] http://www.fas.harvard.edu/~loebinfo/loebinfo/Proportions/humanfigure.html
[20] http://www.wired.com/2012/08/olympics-physics-swimming/




Using the full stroke data from this source[21], we can estimate a cubic line to calculate an estimate of the wave drag coefficient (note that a full stroke regression line will be lower than the arm-only regression line gained from the rest of the data).

| speed | drag | speed-cubed | estimate of k |
|---|---|---|---|
| 1.85 | 30 | 6.331625 | 4.738120151 |
| 1.87 | 28 | 6.539203 | 4.281867377 |
| 2.03 | 31 | 8.365427 | 3.705728351 |
| 2.07 | 36 | 8.869743 | 4.058742176 |
| | | | **4.196114514** |

The average k value is about 4.2.

# Calculating the Final Time

## Dive

We assume the swimmer perfectly preempts the starting horn, meaning they start their dive at t=0.

The diving platform is 0.6m above the flat water surface.[22] Our swimmer is going to have a total diving distance of approximately 4m, as he is 2.1m tall and the additional 1.9m is accounted for due to the momentum of the jump . The diving motion is assumed to have a peak height of 1.5m (initial 0.6m above the surface and 0.9m of gained height due to the jump). This means using the equation $d = \frac{1}{2}at^2$ , $1.5 = \frac{1}{2}at^2$ and a = 9.81 as it is the acceleration due to gravity, to find the time the swimmer hits the water.

This gives a total diving time of **0.55s.**

We are assuming that, following the dive, the swimmer will be at maximum velocity. This means the next parts of our model do not have to take into account the acceleration stage.

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[21] https://www.researchgate.net/figure/228355773_fig12_Figure-13-Individual-estimates-for-wave-drag-dependent-on-speed-Error-bars-indicate-the

[22] https://www.swimoutlet.com/guides/swimming-pool-dimensions

## Dolphin Kick Portions

Earlier, we found the power equation for underwater dolphin kick sections to be as follows:

$$P = \tfrac{1}{2}\rho v^3 C_D A$$

Rearranging this to make v the subject gives:

$$v = \sqrt[3]{\frac{2P}{\rho C_D A}}$$

$$v = \sqrt[3]{\frac{2(1200)}{997 \times 0.4 \times 0.21}}$$

Solving for v gives a velocity of $3.01ms^{-1}$

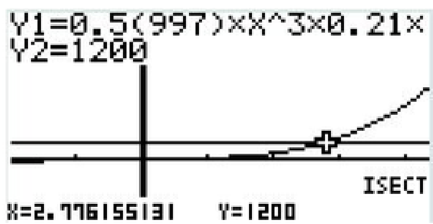
The underwater dolphin kick portions take up a total of 25.6m. This gives a total underwater dolphin time of **8.50s.**

## Front Crawl Portions

Earlier, we found the power equation for surface sections to be as follows:

$$P = \tfrac{1}{2}\rho v^3 A\,(C_L sin\,\theta + C_D)\; + \; kv^4$$

$$1200 = \tfrac{1}{2}(997)v^3(0.21)\,(0.2 sin\,7 + 0.4)\; + \; 4.2v^4$$




This can be solved graphically, giving a velocity of $2.78ms^{-1}$ with all constants substituted.

The front crawl portions take up a total of 68.6m. This gives a total front crawl time of **24.7s.**

## Turn

As with the dive, we are assuming the swimmer will be at maximum velocity following the turn. We are also assuming the power output is equal to the product of velocity and the Kutta–Joukowski lift due to the magnus effect.

Calculating Kutta-Joukowski lift (assuming the turn can be approximated by a circle with a 0.4m radius):

$$F_{turn} = L\rho v 2\pi r^2 \omega$$

Substituting for $\omega$ (and then values: L= width of swimmer = 0.6m) gives:

$$F_{turn} = (L\rho v 2\pi^2 r^2)/t$$

$$F_{turn} = L\rho v^2 \pi\, r$$

$$F_{turn} = 751.7v^2$$

Multiplying the total lift force during the turn by the velocity of the swimmer will thus be equal to the total power output of approximately 1200W.

$$P = F_{turn}v$$

$$1200 = 751.7v^3$$

This gives a turn velocity of $1.17ms^{-1}$. When dividing the distance covered in the turn, $0.4\pi$, by this velocity, a total turn time of **1.10s** is obtained

## Final Time

Dive time = 0.55s

Underwater dolphin time = 8.50s

Front crawl time = 24.7s

Turn time = 1.10s

Giving a total time of **34.8s** for the Olympic 100m freestyle event**.**

# Conclusion and Limitations of the Model


[23]The time of 34.8s is fairly reasonable given that we are dealing with an optimum human specimen, who utilises his power output close to 100% efficiently. In reality, no swimmer has such a technique - humans are inherently inefficient in the water and the basic freestyle front crawl and dolphin kick (techniques we have assumed are utilised here) are nowhere near to 100% efficient. Even more significantly, more advanced swimming technique (and even with the benefit of performance enhancing drugs) would not take us near the complete efficiency of our human.

As a result, our swimmer somewhat stretches the meaning of 'humanly' possible. Humans are very inefficient in the water compared to other animals[24], and it seems unlikely that the time of 34.8s is possible within current human parameters. However, in

[23] http://www1.pictures.gi.zimbio.com/Olympics+Day+8+Swimming+-E0XR6Fsw_xl.jpg

[24] https://news.google.com/newspapers?nid=1955&dat=19740918&id=ZAcrAAAAIBAJ&sjid=XZgFAAAAIBAJ&pg=1442,4309628&hl=en

the future, with the advent of human genetic modification, it is possible that a 'human' can be produced who can more efficiently swim through water, whilst still being considered technically human (i.e able to interbreed with other humans to produce fertile offspring)[25].

It is important to note that the power level of 1200W provided in our calculation does not accurately represent the power output of the ideal swimmer throughout the 100m. There is uncertainty within the 1200W figure itself, and is subject to variation from individual physiological differences, as well as the fact that inevitably, the power output may be greater at the start of the 100m race compared to near the end. This is because near the end of his race, the swimmer is more likely to experience physical and mental fatigue[26]. We assume that the swimmer paces himself perfectly and progresses through each section at a perfectly uniform speed save for the dive and the turn.

Following the dive and the turn, we assume that our swimmer is at their maximum velocity, and no acceleration or deceleration takes place during the dolphin and front crawl portions themselves. We also assume the deceleration from the dolphin kick portion to the slower front crawl portion is instant and this change in velocity is not taken into account.

In addition, we assume that the swimmer travels in a straight perfect line, with no horizontal deviations from the shortest route. While this has a relatively small impact on his overall time, deviation from the straight path could slightly increase the distance that the swimmer ends up travelling.

Information on wave drag was relatively sparse, so a very simple cubic model was utilised. The value of k was furthermore determined by only four points which had fairly large error bars. It is difficult to know if there would have been a significant difference in wave drag had we modelled it in a more complex manner. Information on lift drag with regard to this situation was relatively sparse also, but the effect of this was found to be largely negligible in the end.

It is also assumed that the ideal swimmer has very high lung capacity and $VO_2$ max[27] figure. This would allow the athlete to minimise the number of times required to surface to take a breath, and it is also assumed that this action does not have a discernable impact on the overall velocity of the swimmer. Given the 100 metre swimming race is a short, high-intensity exercise, anaerobic respiration would be prioritised over aerobic respiration, reducing the athlete's need for oxygen. Therefore, our assumption that the athlete's breathing motion has very little impact on his time is reasonable.

Additionally, the ruleset of the Olympic games is subject to change. It is possible that in future the allowed swimwear becomes less strict, giving smaller drag coefficients. Furthermore, the 15m rule was not always in place.[28] It is possible that this rule could change at some point, meaning that larger portions of the course could be done at faster speeds using dolphin or fish kicks underwater, enabling faster times to be achieved.

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[25] http://www.dictionary.com/browse/species
[26] http://coachsci.sdsu.edu/swim/bullets/46aFATIGUE.pdf
[27] https://en.wikipedia.org/wiki/VO2_max
[28] https://www.kiefer.com/blog/15-meter-resurfacing-marker-underwater-swimming-rule

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