

Report 1089

What is the fastest humanly possible time for the Olympic 100m freestyle event?

Summary

The Olympic Swimming Events are some of the most prestigious and exciting areas of the Olympic Games, attracting exceptionally talented athletic men and women. Over the many years since swimming was introduced to the Games, records have decreased with improvements in training, equipment and performance.

By examining the different components of this race we can determine how fast the most effective athletes currently the world could perform each section. We came up with two methods of exploring this question: statistical extrapolation and using physical theory. Through extrapolation to find the asymptote of the world record progression, we found a rough predicted time of 42.1 seconds. Statistical predictions are limited due to the nature of the sport, athletic performance and scarcity of data. We limited our computations to athletic abilities that have currently been observed in the world, as the question is posed in the present tense, and as such we regarded humans with abilities not yet genetically viable as not considerable.

By examining the maximum explosive power of our world's current top athletes, we could examine the reaction time, as well as the dive distance and time. Using VO2 max data and the relationship between power and speed in the water, we found that we could improve Michael Phelps' freestyle swimming capability and fitness. Finally, we discovered the average decrease that tumble turns have on swimming times. Combining the components of this race returned a time figure of **44.05 seconds, our final result.**

Introduction

In an Olympic year, it is a near certainty that many records will be broken, and new world standards will be set. Swimming is one of the most established Olympic sports, being first introduced in 1896¹. As such, much attention will surely be placed on it during the Rio Olympic Games this year.

In a sporting world that places more and more emphasis on collected data and mathematical predictions, it is of great interest to predict the minimum possible time that an Olympic swimmer could complete the 100m Freestyle event.

Report

Initial Estimate

A rough estimate was made by extrapolating previous world records² by finding an equation of best fit. An asymptote is expected due to the physical limitations of the human body. This asymptote will provide a rough estimate. Only values from the 1970's onwards were used since the availability of better timekeeping technology means the data is more accurate. Using the online graph fitting software MyCurveFit³, the following equation is determined

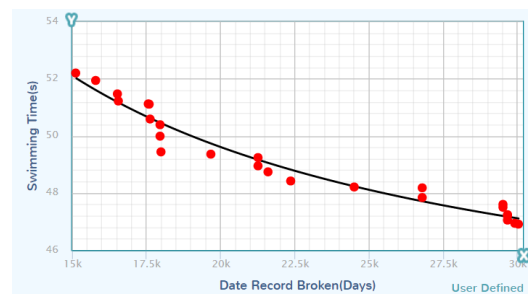


Figure 1: Records vs. Date Set

$$y = \frac{149594.1}{x} + 42.12509$$

The y axis represents the world record (in seconds) and the x axis their relative times in days to an arbitrary point in time. The time from which the times are relative to are irrelevant since the asymptote is found from taking limit as x tends towards infinity.

$$\lim_{x \rightarrow \infty} \left(\frac{149594.1}{x} + 42.12509 \right) = 42.12509$$

Thus an initial estimate for the fastest humanly possible 100m freestyle time is approximately **42.13s**.

Methodology

It was decided that we would partition the 100m race into 5 constituent sections. Given that the pool is 50m long, as per Olympic regulations, the race will occur over two laps of the pool. Firstly, the swimmer must dive into the pool, including some period of reaction time. Then, the swimmer will travel underwater for some period of time, before surfacing and continuing at a steady state speed until the tumble turn at the end of the first lap.

¹ Swimming at the Summer Olympics. Retrieved August 6 2016 from Wikipedia: https://en.wikipedia.org/wiki/Swimming_at_the_Summer_Olympics

² https://en.wikipedia.org/wiki/World_record_progression_100_metres_freestyle

³ www.mycurvefit.com

Following the tumble turn, the swimmer will then travel at the same steady state speed for the rest of the lap.

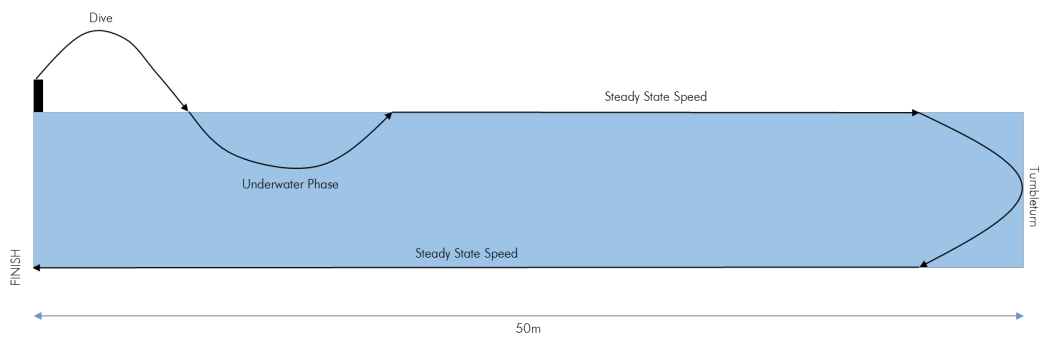


Figure 2: Breakdown of Motion

Dive

Standard Olympic diving blocks are 0.76m above the water level of the pool⁴. Critically, human reaction times must be taken into account. We assessed the time and distance taken to dive through the use of kinematics equations. Firstly, we must calculate the magnitude of velocity with which the diver leaves the diving board.

$$E_K = \frac{1}{2}mv^2$$

$$P = \frac{E}{t}$$

$$P = \frac{mv^2}{2t}$$

$$\Rightarrow \sqrt{\frac{2Pt}{m}} = v$$

Hence, the resultant velocity depends on the time taken to exert power, and the power exerted by the swimmer per unit of mass.

Therefore, if $k = \frac{P}{m}$ (Power per unit mass, in $W kg^{-1}$).

$$\sqrt{2kt} = v$$

It is then important to ascertain the power that a swimmer could generate, their mass, and the time taken for them to leave the blocks after beginning to expend power.

⁴ <http://www.indiana.edu/~ccss/research/Start%20study/effect%20of%20block%20height.php>

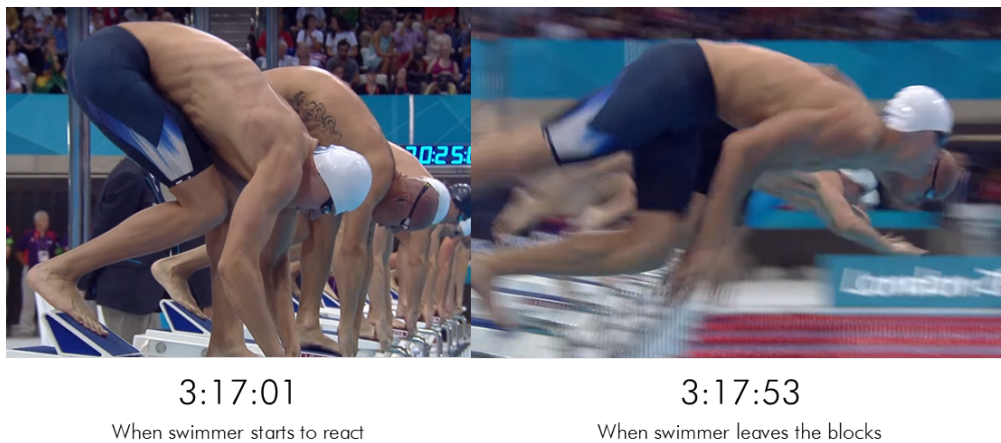


Figure 3: Illustrating the time taken to exert all power to accelerate to maximum speed⁵

Using a slow motion recording of Nathan Adrian's Gold medal race in the London 2012 Olympics 100m Men's Freestyle event, we were able to determine how long he was accelerating for. The time taken for Nathan to start his moving until when his foot left the block (and thus the time when he stopped laterally accelerating) was 0.52s.

Typically, elite swimmers have an 'on-block' time of 0.74s⁶ – defined as the time from when the starter gun goes, to when the swimmer leaves the block. Of course, this includes reaction times. If we factor in reaction times for Nathan Adrian's dive, his total time comes to 0.69s – slightly faster than average. Hence, he took 0.17s to react, and 0.52s to accelerate.

According to FINA, "After all swimmers are "stationary" (SW 4.1), any swimmer who moves before the starting signal may be disqualified when such movement is observed and confirmed by both the starter and referee (SW 2.1.6). When video-tape timing system (FR 4.7.3) is available, it may be used to verify the disqualification."⁷

Therefore, the earliest Adrian could move was exactly when the starting signal was given, meaning that his on-block time could have been a minimum of 0.52s. As such, we may make the assumption that the on-block time could be a minimum of around 0.52 seconds. This may be very unlikely and unrealistic, but the question states what the fastest humanly possible time could be.

Once the swimmer has left the block, they are travelling at a velocity as defined by

$$\sqrt{2kt} = v$$

Hence, as $t = 0.52s$, as per Nathan Adrian's dive, it is now important to ascertain exactly what the highest power to weight ratio of a human could realistically be for this kind of motion.

⁵ Annotated by Team 1089, images from <https://www.youtube.com/watch?v=VO7y41uBdUA>

⁶ <http://theconversation.com/take-your-marks-the-science-behind-the-perfect-swimming-dive-29392>

⁷ <http://www.fina.org/content/sw-4-start>

Data exists detailing Usain Bolt's power expended at the instant of start in the Olympic 100m race. On the start line, Bolt expends $2619.5W$ ⁸, in a very similar motion to springing off the starting blocks in a swimming race. Additionally, with his weight of $92kg$ ⁹, this gives a power to weight ratio of $28.5W/kg^{-1}$. Even extreme weightlifters, some of the most powerful athletes in the world will only just get close to this ratio¹⁰.

Hence, with substitution, we find that the theoretical leaving velocity of our swimmer is:

$$v = \sqrt{2 \times 28.5 \times 0.52}$$

$$v = 5.34ms^{-1}$$

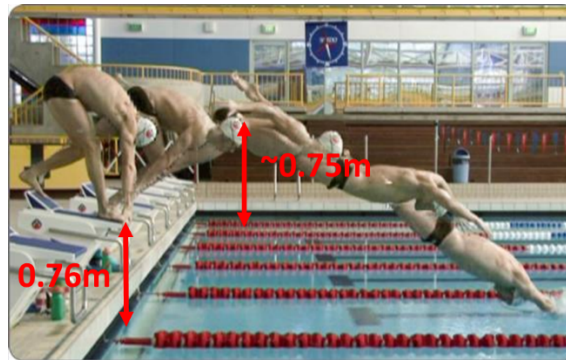


Figure 4: Height of a swimmer leaving the blocks

As per Figure 4, Olympic swimmers seem to dive horizontally from the blocks, maximising their lateral speed. As such, we can make the assumption that the diver is a body that accelerates only laterally, and gravity accelerates them downwards. Assuming that air drag has a negligible effect at these low speeds, the lateral distance travelled by the diver will be:

$$s_h = 5.34t$$

Where t is the time taken for the diver to make impact with the water. It can be calculated through:

$$s_v = ut + \frac{1}{2}at^2$$

$$s_v = 0t + \frac{1}{2}(-9.81)t^2$$

$$s_v = \frac{1}{2}(-9.81)t^2$$

Hence, s must be calculated. As previously stated, the blocks of a swimming pool are 0.76m above the water level of the pool. In addition, we assume the centre of mass of the swimmer to be a further 0.75m above this, based on image analysis of a swimming diver as

⁸ <http://www.independent.co.uk/news/science/revealed-the-science-behind-usain-bolt-power-burst-used-to-the-100m-world-record-8732545.html>

⁹ http://www.diffen.com/difference/Carl_Lewis_vs_Usain_Bolt

¹⁰ <https://www.youtube.com/watch?v=61dhC4erQPQ>

displayed in Figure 4. This measurement is taken at the last moment the swimmer is in contact with the diving block, and therefore the last moment that the swimmer is accelerating. Swimmers tend to be taller than average¹¹, and so we consider this to be a reasonable assumption. Ideally, a swimmer would spend more time in air because air provides less fluid resistance than water¹² and is thus more efficient to travel through. Hence, the total displacement that the swimmer will travel through vertically is -1.51m (this is the value of s , negative because they are travelling downwards).

$$-1.51 = \frac{1}{2}(-9.81)t^2$$

$$t = 0.55\text{s}$$

Employing this value of t , with substitution we find:

$$s_h = 5.34 * 0.55$$

$$s_h = 3.0\text{m} (2 \text{ s. f.})$$

Hence, the dive covers 3.00 meters, and takes 1.07s including the 'block time' and the air time.

Underwater Phase

The underwater kicking phase is utilised to lengthen the deceleration phase from 5.34 ms^{-1} to the steady state speed of the swimmer. Assume constant deceleration for the underwater phase for simplicity. As the deceleration is uniform the average speed will be $\frac{v+u}{2}$ or $\frac{5.34+2.1}{2}$. This results in a velocity for the underwater phase of 3.72 ms^{-1} . As the underwater kicking must take our swimmer from 3.00 to 15.00 metres (the maximum underwater limit set by FINA)¹³, this will take 3.23 seconds.

Maximal Steady State Speed

We assume that the swimmer will remain at a steady state speed between the Underwater Phase, the tumble turn and after the completion of the tumble turn until the finish. This is the most efficient way to swim because any increase in velocity requires a cubic increase in power output, as per the Drag Equation:

$$F_D = \frac{1}{2}\rho v^2 C_D A \quad 14$$

Hence, it is inefficient to vary the speed because the power required is much more than proportional, and thus will tire the swimmer out more than going at a steadier speed.

Power is related to Force through:

¹¹ https://en.wikipedia.org/wiki/Height_in_sports

¹² <http://www.explainthatstuff.com/aerodynamics.html>

¹³ <https://www.kiefer.com/blog/15-meter-resurfacing-marker-underwater-swimming-rule>

¹⁴ <https://www.grc.nasa.gov/www/k-12/airplane/drageq.html>

$$P = Fv$$

Therefore, power lost due to drag is:

$$P_D = \frac{1}{2} \rho v^3 C_D A$$

Steady state speed will be reached when the power output of the swimmer is equal to the energy lost per second to fluid drag.

$$P_S = P_D = \frac{1}{2} \rho v^3 C_D A$$

Where P_S is the power generated by the swimmer.

In order to work out how fast a human could conceivably swim at a steady state for, we will compare a known elite swimmer's time, and then calculate how much faster they could possibly swim through a metric known as VO_2 MAX. VO_2 MAX is a measure of the maximum volume of oxygen that an athlete can metabolise under full aerobic load. It is measured in millilitres per kilogram (body mass) per minute (ml/kg/min).

Hence, for example, take Michael Phelps. The fastest time that he has swum 100m in is 47.3 seconds¹⁵. Upon reviewing his race, using slowed down camera footage, we calculated that he was able to swim 25 meters freestyle in 13.0 seconds during that 100m race¹⁶. Assuming this pace was carried throughout all of the steady state sections of the race (as per our categorisation as shown in Figure 1), this gives Phelps an average steady state speed of 1.92 ms^{-1} , or 6.92 kmh^{-1} .

This velocity may now be substituted into the Drag Equation to calculate Phelps' theoretical power output. However, values for ρ , C_D and A must first be found before we are able to solve the equation. The drag coefficient, A , is 0.36 for most elite swimmers¹⁷. Chlorinated pool water has a fluid density coefficient of 992.7 $kg\ m^{-3}$ ¹⁸. Frontal area for competitive male swimmers is typically around 0.23 m^2 ¹⁹.

This means that Phelps' power output can be calculated to be:

$$P_S = \frac{1}{2} (992.7 * 1.92^3 * 0.36 * 0.23)$$

$$P_S = 290W \text{ (2 s. f.)}$$

¹⁵ <https://swimswam.com/how-fast-will-michael-phelps-swim-in-2016-gold-medal-minute-presented-by-swimoutlet-com/>

¹⁶ <https://www.youtube.com/watch?v=HEonM3jqDF0>

¹⁷ Barbosa T., Morais J., Goncalves J, Marinho D., Silva A. (2013) A Swimmer's Classification System based in kinematics, anthropometrics and hydrodynamics.

¹⁸ cssf.usc.edu/History/2011/Projects/J0117.pdf

¹⁹ https://www.researchgate.net/publication/269175068_Planimetric_frontal_area_in_the_four_swimming_strokes_Implications_for_drag_energetics_and_speed

Hence, Phelps' power output while swimming over this time period would be around 290W. This is considerably less than during the dive, but is also spread over a much longer time (circa. 40 seconds).

Interestingly, the relationship between VO_2 MAX and Power is directly proportional²⁰. Therefore, we can state that:

$$[VO_2] \propto P_S = \frac{1}{2} \rho v^3 C_D A$$

Michael Phelps has a VO_2 MAX of $76 \text{ ml kg}^{-1} \text{ minute}^{-1}$ ²¹. The highest ever recorded VO_2 MAX was

$97.5 \text{ ml kg}^{-1} \text{ minute}^{-1}$ by Oskar Svendsen²². Because power is directly proportional to VO_2 MAX, we can employ a multiplier to estimate what our theoretical swimmer might be able to produce, given the physical attributes of Phelps and the VO_2 MAX of Oskar Svendsen.

$$\frac{97.5}{76} = 1.28 \text{ (Multiplier)}$$

Therefore, we can multiply Phelps' power by this figure to calculate the theoretical ideal swimmer's steady state speed.

$$290W * 1.28 = 370W$$

Therefore, our theoretical swimmer would be able to output 256W for the duration of the race. If we convert this back into velocity, through our prior equation:

$$P_S = \frac{1}{2} \rho v^3 C_D A$$

$$370W = \frac{1}{2} (992.7 * v^3 * 0.36 * 0.23)$$

$$v = 2.1 \text{ ms}^{-1}$$

Therefore, our ideal swimmer would be able to travel at 2.1 ms^{-1} for the duration of the race.

Tumble Turn

The method by which a swimmer turns around at the end of each lap is crucial to achieving a fast time. By converting their linear momentum to rotational momentum and back to linear momentum going the other way with minimal loss of energy, a swimmer can minimise time lost turning around. As we did not have the necessary equipment or data to accurately model the energy transfer of a tumble turn, we made the assumption that the time gains through the use of a tumble turn is entirely due to technique, and any added physical capabilities would not provide any further gains. We also assumed that the holders of the American and World records are of a sufficiently high level that there can be little

²⁰ www.ncbi.nlm.nih.gov/pubmed/7960315

²¹ https://prezi.com/hzt8eclvorq_/lochte/

²² <http://www.topendsports.com/testing/records/vo2max.htm>

further improvement on their technique. We obtained data from the US Swimming database, comparing the records of both short course (25m laps) and long course (50m laps), where the difference in times should solely be due to the inclusion of tumble turning.

| | Long Course | Short Course | Difference | Number of turns difference | Average gain per turn | Average |
|-------------------|-------------|--------------|------------|----------------------------|-----------------------|----------|
| 50m World | 20.91 | 20.26 | 0.65 | 1 | 0.65 | |
| 50m US | 21.37 | 20.85 | 0.52 | 1 | 0.52 | |
| 100m World | 46.91 | 44.94 | 1.97 | 2 | 0.985 | |
| 100m US | 47.33 | 46.25 | 1.08 | 2 | 0.54 | |
| 200m World | 102 | 99.37 | 2.63 | 4 | 0.6575 | |
| 200m US | 102.96 | 101.08 | 1.88 | 4 | 0.47 | |
| 400m World | 220.07 | 212.25 | 7.82 | 8 | 0.9775 | |
| 400m US | 222.78 | 214.85 | 7.93 | 8 | 0.99125 | 0.723906 |

From this data, we determined that for each tumble turn in a race, a swimmer will go 0.72s faster. Thus in the Olympic race where our swimmer is competing, the inclusion of one tumble turn will decrease his time by 0.72s faster than if he has swum the full 85m after the underwater kick section at his steady state speed. Thus by adding the time taken to travel through the air, the time spent underwater kicking, and the 85m at steady speed, and subtracting the 0.72s, the swimmer's final result is gained.

Evaluations & Further Assumptions

Swimmer:

- The swimmer has the max power output of Usain Bolt for the jump off the starting block.
- The shoulder cross sectional area of the swimmer is 0.23m^2 .
- All physical capabilities will be within the range of natural human ability, without the use of performance enhancing stimuli.
- The swimmer's drag coefficient is constant at 0.36^{23}

Start Phase:

- Air drag while jumping is negligible
- Acceleration due to gravity is 9.81ms^{-2}
- The swimmer predicts the start siren and begins moving as soon as it goes, negating reaction time error.
- Swimmer's power can be approximated by Bolt's on start as nearly all power is generated by legs.
- Starting platform is 0.76m above the water
- Centre of mass gains 0.75m in height after start before falling

²³ Barbosa T., Morais J., Goncalves J, Marinho D., Silva A. (2013) A Swimmer's Classification System based in kinematics, anthropometrics and hydrodynamics.

Underwater kicking Phase:

- There is no glide phase between water entry and underwater kicking phase.
- The swimmer decelerates at a constant rate during this phase.

Steady State Swimming Phase:

- The swimmer travels at the same speed, as a constant speed results in a faster time.²⁴

Tumble Turn Phase:

- Technique has been nearly perfected and cannot be significantly improved.
- Physical ability has little effect on a tumble turn.

Pool:

- The pool is exactly 50.00m long, requiring only a single tumble turn in the race.
- The pool has a fluid density of 992.7kg m^{-225} and is completely flat, with no turbulence
- Pool is 28°C in accordance with FINA guidelines²⁶
- The race occurs at an Olympic event and thus follows all Olympic and FINA guidelines.

Conclusion

The total time for our dive section is 1.07 seconds, over 3m of distance. Our time then spent underwater dolphin kicking back to the surface is 3.23 seconds taking 12 meters. Then, 85 meters is spent swimming afterwards, at a steady state speed of 2.1ms^{-1} , taking 40.48 seconds. On average, a tumble turn cuts 0.72 seconds per turn from the time otherwise at steady state speed.

Hence, total time:

$$\text{Total Time} = 1.07 + 3.23 + 40.48 - 0.72$$

$$\text{Total Time} = 44.06\text{s}$$

Comparison to Graphical Predictions

Initially, we predicted graphically that the fastest humanly possible 100m freestyle time was approximately 42.13 seconds. However, upon analysing the human ability and movements of swimming mathematically and not statistically, we reached a conclusion that the theoretical fastest possible time for a human to swim the Olympic 100m freestyle event is 44.06s. Hence there is a 1.93 second discrepancy. However, this is expected, as the data we had to statistically model the record times was limited in the size of the dataset.

²⁴ http://www.flatlandswimming.ca/canfs/__doc__/136185_2_RacePacing.pdf

²⁵ cssf.usc.edu/History/2011/Projects/J0117.pdf

²⁶ <http://www.fina.org/content/fr-2-swimming-pools>