# Auckland Mathematical Olympiad

## Problems

1. A single section in a stadium can hold either 7 adults or 11 children. When $N$ sections are completely filled, an equal number of adults and children will be seated in them. What is the least possible value of $N$?

2. Triangle $ABC$ of area 1 is given. Point $A'$ lies on the extension of side $BC$ beyond point $C$ with $BC = CA'$. Point $B'$ lies on extension of side $CA$ beyond $A$ and $CA = AB'$. $C'$ lies on extension of $AB$ beyond $B$ with $AB = BC'$. Find the area of triangle $A'B'C'$.

3. In a square of area 1 there are situated 2024 shapes whose total area is greater than 2023. Prove that they have a point in common.

4. Which digit must be substituted in place of the star below so that the following large number is divisible by 7?

$$\underbrace{66\cdots66}_{2023}\star\underbrace{55\cdots55}_{2023}$$

5. There are 11 quadratic equations on the board, all with undetermined coefficients and constants. Each quadratic equation looks like this:

$$\star x^2 + \star x + \star = 0.$$

Two players are playing a game making alternating moves. In one move each of them replaces one star with a real nonzero coefficient or constant.

The first player tries to make as many equations without roots while the second player tries to make their number as small as possible.

What is the maximal number of equations with roots that the first player can achieve if the second player plays to her best?

Describe the strategies of both players.

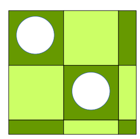
6. Suppose that a machine has an infinite sequence of lights numbered $1, 2, 3, \ldots$, and that you know the following two rules about how the lights work:

   - If the light numbered $k$ is on, the lights numbered $2k$ and $2k+1$ are also guaranteed to be on.
   - If the light numbered $k$ is off, then the lights numbered $4k+1$ and $4k+3$ are also guaranteed to be off.

   Suppose you notice that light number 2023 is on. Which other lights are guaranteed to be on?

7. Each square on an 8 by 8 checkers board contains either one or zero checkers. The number of checkers in each row is a multiple of 3, while the number of checkers in each column is a multiple of 5.

   The top left corner of the board is shown below, so how many checkers are there on the board in total?



8. How few numbers is it possible to cross out from the sequence

$$1, 2, 3, \ldots, 2023$$

   so that among those left no one number is the product of any two (distinct) other numbers?

9. Quadrilateral $ABCD$ is inscribed in a circle with centre, $O$. Its diagonals, $AC$ and $BD$, are perpendicular. Prove that the distance from $O$ to $AD$ is half the length of $BC$.

10. Find the maximum of the expression,

$$||\ldots||x_1 - x_2| - x_3| - \ldots | - x_{2023}|,$$

   where $x_1, x_2, \ldots, x_{2023}$ are distinct natural numbers between 1 and 2023.