Team 1049

"What is the largest payload that can be launched into orbit by slingshot"

Summary:

In this report, we aimed to calculate the maximum payload able to be launched into orbit by satellite. To do this, we needed to maximise the initial velocity to reach the final velocity required for orbital speed, whilst minimising the energy lost through the conversion of elastic potential energy to kinetic energy. Our final mass calculated was approximately 32,800kg. We mainly considered the limits of human technology to build the most powerful spring based slingshot system possible, and the minimum speed that must be reached for the mass to be launched into orbit.

1. Introduction:

Launching any type of spacecraft into a low-orbiting flight path typically requires an exorbitant amount of fuel to create the necessary thrust required for take-off. The resulting emissions from burning this fuel are released into the atmosphere with obvious negative environmental consequences - which will inevitably increase as space exploration continues to reach new heights. Therefore, exploring alternative launch technologies to transport manmade objects into orbit is a vital step to improve the sustainability of aerospace engineering.

Slingshots are an example of a possible alternative launch technology, but the efficiency of their use (thus the payload they can support) is limited by their ability to launch a mass with sufficient velocity to reach the height required to enter the Earth's orbit. This means it is paramount to accurately model the maximum payload that can be launched into orbit by a slingshot, in order to determine their viability as a future component of space accessibility. Furthermore, the effectiveness of slingshots is also limited by air resistance from the object launched and the breaking point for the spring system comprising the system.

1.1 Definition of Key Terms:

We define the 'payload' *(Merriam-Webster. (n.d.). Payload. Retrieved 06/08/2022, from* https://www.merriam-webster.com/dictionary/payload) as the load carried by a vehicle that is exclusive of what is necessary for its operation. In terms of a typical spacecraft, the payload does not include the mass of fuel, fuel tanks, engines, propellants or the materials required to build the spacecraft's exterior shell. Therefore, we have defined the 'largest payload' as the greatest mass able to be carried by our vehicle that does not exceed the maximum mass that the slingshot system can support.

We define the 'vehicle' as the mechanism that our payload is contained within in order to safely reach orbit. We have decided that our vehicle should mimic the shape of a typical rocket due to its proven aerodynamic characteristics, and should include the properties of a slim body with nose cone and fins (for decreased air resistance and increased stability/control). As this vehicle is being launched by slingshot, it does not require its own source of fuel or propulsion - therefore does not need to be carrying fuel, tanks, engines or propellants. This means the vehicle is simply the shell of a rocket, so therefore the payload can make up the entirety of the total mass (aside from the mass of materials required for the vehicle body). We found that the mass of the shell of a typical rocket comprises 3-4% of the total mass, meaning that the payload of our vehicle can make up 96-97% of the maximum mass that can be launched by our satellite. We have also decided that the body of our vehicle will be made of 6061-Aluminium, which is the lightest yet strongest aerodynamic material (weighing 9.667kg per 4x12ft sheet).*(What kinds of materials are used for rockets? Retrieved 06/08/2022, from* https://howthingsfly.si.edu/ask-an-explainer/what-kind-materials-are-used-rockets https://www.sciencelearn.org.nz/resources/392-rocket-aerodynamics)

We are defining 'orbit' as when the object stays in motion around earth. We have chosen an orbit height of 200km above Earth, as the lowest satellite altitude recorded was Japan's Tsubame satellite launched in 2019, which orbited at a lowest height of 167.4km.* Therefore we can be confident that this orbit height is achievable and possible, while being very low. Minimising the orbital height will mean that the vehicle and payload has a reduced vertical distance to cover, meaning that the impact of air resistance on velocity will also be minimised (thus allowing the load to reach orbit more easily and further justifying our removal of air resistance in velocity equations).
(Japan's Tsubame records lowest ever satellite altitude. Retrieved 06/08/2022, from https://www.spacetechasia.com/japans-tsubame-records-lowest-ever-satellite-altitude/#:~:text=Tsubame%2C%20an%20Earth%20Observation%20satellite,an%20altitude%20of%20167.4%20km)

We have defined 'slingshot' as a system in which extension springs can be used to draw back our vehicle (and payload) and then release it at a particular angle with the required velocity to reach orbit. Discussed further below, under 2.3 Slingshot Model.

1.2 Initial Assumptions:

- *This report uses a number of reasonable assumptions to reduce the complexity of modelling required to answer the question.*

We have made the assumption that the vehicle will not require any thrusters or engine to launch it into orbit. Therefore, fuel is not required in the calculation of the total mass, as the slingshot will produce the force necessary for the rocket to enter the earth's orbit.
We are also assuming that the only components of our vehicles' mass will be the mass of materials required for the shell, plus the mass of the payload - thus if the mass of the shell of a typical rocket makes up 3-4% then the payload can make up the remaining 96-97%.

Furthermore, we have assumed that air resistance is negligible as the angle we chose allows for horizontal distance travelled to be minimised. Our vehicle design will also be as aerodynamic as possible which would further reduce air resistance, thus justifying our decision to omit it from calculations.
Through the minimisation of air resistance, we also assume that no energy is lost in the launching of the rocket. Therefore, Ep stored in slingshot = Ek transferred to vehicle.
Our next assumption was that cost is irrelevant in the creation of the slingshot and vehicle. Therefore, we are not limited by the cost of materials required to create our slingshot system.

2. Calculations and Models

2.1 Orbital Velocity

For any object to be in orbit it must travel at orbital velocity, which is the velocity required for the inertia of the object - its tendency to continue in motion - to be balanced with the force due to gravity acting on the object.* To calculate the orbital velocity required, we are assuming that the vehicle follows circular motion once in orbit, despite the path of orbit around earth being elliptical.
*(How Satellites Work. Retrieved 06/08/2022 from, from*https://science.howstuffworks.com/satellite6.htm#:~:text=Orbital%20velocity%20is%20the%20velocity,150%20miles%20(242%20kilometers).)

(Launching Satellites. Retrieved 06/08/2022 from, https://www.sciencelearn.org.nz/resources/272-launching-satellites)
For a satellite moving in circular motion around earth, the centripetal force is provided by Earth's gravity. Therefore we combined Newton's Law of Gravitation and the equation for centripetal force, in order to calculate the orbital velocity for our chosen altitude of 200km.

F_g = Force due to gravity
G = gravitational constant = 6.67×10^{-11} Nm^2kg^{-1}
M_1 = mass of earth = 5.97×10^{24} kg
m_2 = mass of satellite/rocket
r = distance between centres of each mass
= radius of earth + distance above earth
= 6370,000 + 200000
= 6.57×10^6 m
F_c= centripetal force

Newton's law of gravitation | Centripetal Force

$$F_g = G\frac{m_1 m_2}{r^2} \qquad F_c = \frac{m_2 v^2}{r}$$

$$F_g = F_c$$

$$G\frac{m_1 m_2}{r^2} = \frac{m_2 v^2}{r}$$

$$\frac{G m_1 m_2 r}{m_2 r^2} = v^2$$

$$v = \sqrt{\frac{G \times m_1}{r}}$$

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6.57 \times 10^6}}$$

$$= 7785.157 \ ms^{-1}$$

$$= 28{,}030 \ kmh^{-1}$$

Therefore the final orbital velocity our vehicle must reach is 28,000 km/h (or 7785m/s), which is accurate to two significant figures for an orbit with an altitude of 200km.

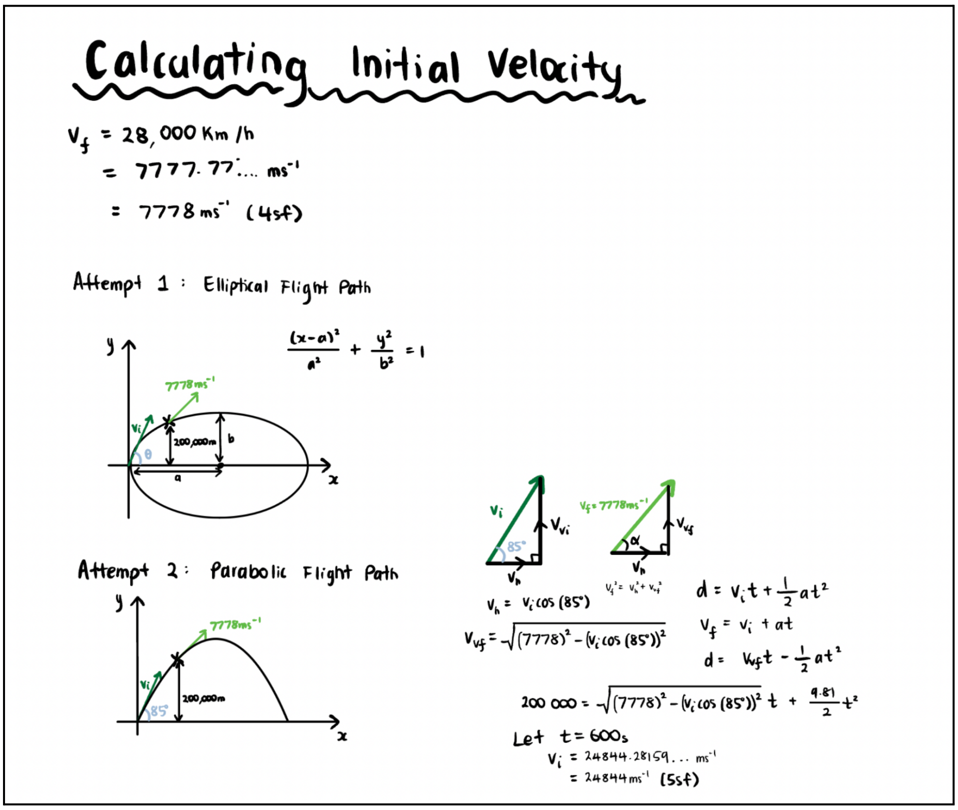
2.2 Modelling Flight Path to Find Initial Velocity Required:

Initially, we attempted to model the launch of our vehicle after a Gravity Turn Trajectory *(Gravity Turn Rocket Trajectories Explained | Rocket Trajectories 4. Retrieved 06/08/2022, https://www.youtube.com/watch?v=VajZiY72Pf0)*

Gravity turn is typically used to launch rockets into space/orbit. Rockets begin with a vertical ascent, before a 'pitch over manoeuvre' is performed, in which some of its thrust is directed horizontally. As the rocket's velocity is no longer directed exactly vertically, downwards force due to gravity is now acting at an angle to the velocity, pulling the rocket towards the earth as it travels. This creates an elliptical flight path due to the curved shape of the Earth.

As our vehicle will have no engines or thrusters so we know our initial velocity must be greater than the required orbital velocity (28000km/h), so that this orbital velocity will be reached at a height of 200,000m despite gravity decelerating the rocket. A lack of engine also removes the ability to perform the pitch over manoeuvre, so our launch must begin at an angle so that a gravity turn will occur. We chose a launch angle of 85° to result in a near-maximum vertical velocity while being under 90°.

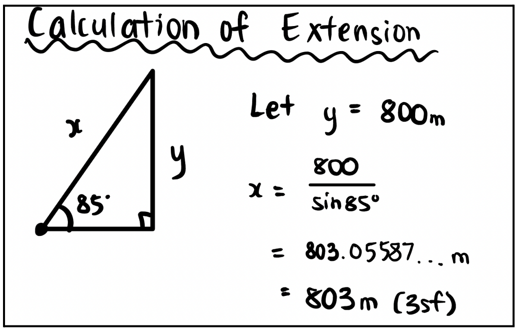
The initial attempt to use an elliptical flight path model led to inconclusive results as we were not able to determine two variables (a and b) in the equation, so we then attempted using a parabolic model. This allowed us to create a formula involving both Vi (initial velocity) and t (time) however, we could not calculate the time based on the information we had. Therefore, based on further research *(NASA Direct. Retrieved 06/08/2022 from, https://www.nasa.gov/mission_pages/shuttle/shuttlemissions/sts121/launch/qa-leinbach.html#:~:text=It%20takes%20the%20shuttle%20approximately,a%20ride%20for%20the%20astronauts.)*, we decided on a flight time of approximately 10 minutes which allowed us to calculate the initial velocity based on kinematic equations as follows:



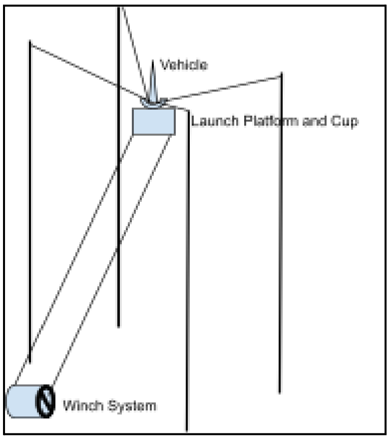
Result:
vi =89,438km/h

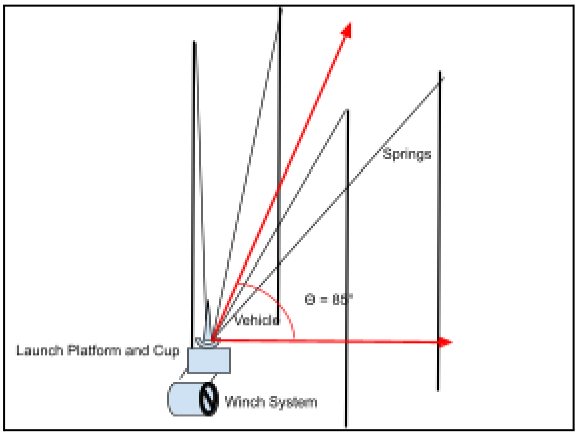
2.3 Slingshot Model

Our slingshot consists of four 800m vertical poles (just under the height of the Burj Khalifa, the current tallest building in the world to ensure this design is technologically possible) with four 400m springs attached, made of high carbon steel, which has one of the highest values for G(Shear modulus of material) of 11.5Nm^6, resulting in a high spring constant. The vehicle will sit on a 'launch cup' which will be attached to a 'launch platform'. This launch platform will be wound downwards along a sloping surface with a winch system, extending 803m (thus giving a spring extension of 400m), where the cup will then detach from the platform and launch the vehicle at an angle to the ground of 85° with an initial velocity of 89,438km/h as previously calculated. See Figures B and C below for slingshot design and Figure A for initial velocity of vehicle at launch.



We assumed that our winch can carry a maximum load of approximately 45,359kg, based on the world record for maximum weight pulled by a winch. Therefore, in our calculations, we allowed for a certain mass to be assigned for the mass of the springs and the launch cup. Due to this, while placing more springs in parallel to our current ones would allow us to increase the spring constant and increase the total mass, we were unable to do this due to the limitations of our winch.



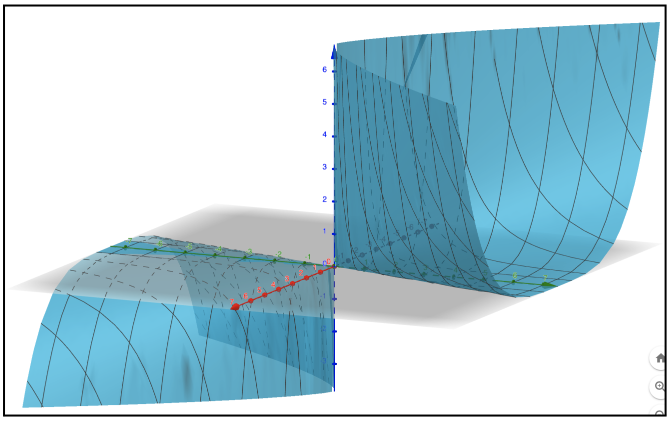


3D Modelling:

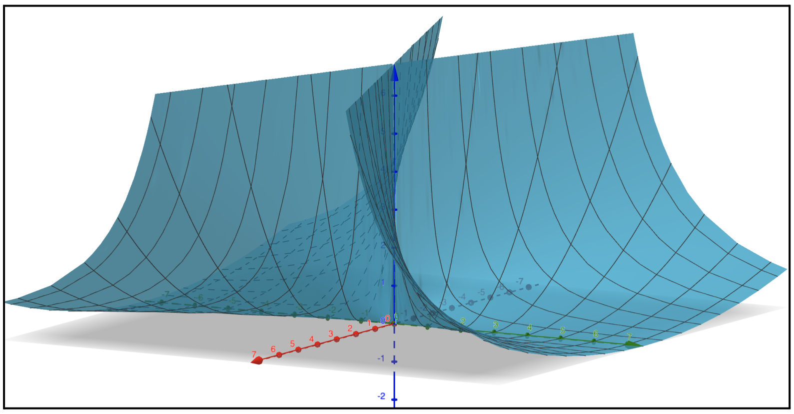
The graph below represents the multivariable function, f(x,y)=x^4/y^3, with x = d, and y = ND. The value of the spring constant is dependent on four key factors:

1. Wire diameter
2. Coil diameter
3. Free length
4. Number of active coils

https://www.acxesspring.com/extension-spring-design-guide.html



Therefore, in order to maximise the spring constant, either the wire diameter (d) must be maximised and the product of the number of active coils (N) and the mean diameter of the spring (D) must be minimised. However, the calculations below in which the partial derivatives were taken with respect to x and y indicated that the maximum value for the spring constant occurs when both x and y are equal to zero. In our context this is not feasible as this would mean the spring would not exist. Therefore, we instead therefore researched the feasible parameters for the wire diameter (d) and the mean length of the spring (D).



The graph to the left is the function f(x,y) = x^2/y^2. This is utilised to model the limitations on the spring constant based on the equations for the values of D and d in the calculations below, with x=d and y=D, and f(x) = k. Similar to the case above, in this case the maximum value for the spring constant also occurred when both x and y were zero, which is not feasible. Therefore, the maximum spring constant would occur when x was at the maximum value and y was minimised. Thus, the outer diameter was investigated and determined to be logistically made to be 1m, and the minimum inner diameter as 0.01m. (Inner Diameter = ID, and Outer Diameter = OD).

2.4 Final Calculation of Mass

Based on our assumption of energy conservation we created a formula for the weight of the vehicle and load based on k (spring constant), x (extension of springs) and v_i.

Where G is the shear modulus of material, d is the wire size (m), N is the number of active coils, and D is the mean wire diameter (m).

Calculation of Spring constant

$$K = \frac{Gd^4}{8ND^3}$$

$$N = \frac{\text{Length of spring without hooks}}{d} - 1$$

$$d = \frac{OD - ID}{2} \qquad D = \frac{OD + ID}{2}$$

Let the Length of spring without hooks = 400m

$OD = 1m \qquad ID = 0.01m$

$\therefore d = 0.495 \qquad D = 0.505m$

$G = 11.5 \times 10^6$

$$\therefore K = \frac{(11.5 \times 10^6) \times (0.495)^4}{8 \times \left(\frac{400}{0.495} - 1\right) \times (0.505)^3}$$

$= 830.3046..... \ Nm^{-1}$

$= 830 Nm^{-1}$ (3sf)

Conservation of Energy

$$E_K = \frac{1}{2} mv^2 \qquad E_p = \frac{1}{2} kx^2$$

(elastic)

$$\frac{1}{2} mv^2 = \frac{1}{2} kx^2$$

$$mv^2 = kx^2$$

Let $v = v_i$

$$m = \frac{kx^2}{v_i^2}$$

Attempt to Maximise Spring Constant

$$K = \frac{Gd^4}{8ND^3}$$

Let $d = x$ and $y = ND$

$$f(x, y) = \frac{x^4}{y^3}$$

$$f_x = \frac{4x^3}{y^3} \qquad f_y = -\frac{3x^4}{y^4}$$

$f_x = 0$ when $x = 0$ $(y \neq 0)$ $\qquad f_y = 0$ when $x = 0$ $(y \neq 0)$

$\therefore$ Critical points occur at $(0, 0)$

this is not feasible in this context

Calculation of Mass

Spring Load (F) = kx + initial tension

$x = 803 - 400$

$= 403$

$F = (830 \times 403)$ + initial tension

$= 334490$ + initial tension

$$m = \frac{334490 + \text{initial tension}}{9.81}$$

$$= 34096.83996.... + \frac{\text{initial tension}}{9.81}$$

mass of payload = $0.97m$

$= 33073.93476.... + \frac{97}{981}$ initial tension

$\approx 33{,}073$ Kg (5sf)

F = kx

Where F is the force (N), k is the spring constant (N/m), and x is the extension of the spring (m).

F = ma

Where F is the force (N), m is the mass (kg), and a is the acceleration (m/s^2).

Discussion (Including Limitations):

The use of a parabolic model for the flight of the vehicle (rather than the true elliptical flight path that would occur due to the curvature of the Earth) causes our initial velocity to be slightly inaccurate. However, based on the information we had we were unable to calculate key variables within the equation $x^2/a^2 + y^2/b^2 = 1$.

3. Conclusion:

Conclusively, based on our research and data finding, the largest payload able to be launched into orbit by a slingshot is 32,800kg. In comparison to current rockets which have been launched into orbit this is relatively low (data found the average mass of a rocket to be approximately 100,000kg). (*Rocket Weight. Accessed 06/08/2022 from, https://www.grc.nasa.gov/www/k-12/rocket/rktwt1.html*) However, this was to be expected as our slingshot design has no supporting force from thrusters or engines, as well as no mass percentage of fuel.

Based on the journey required to reach our end goal, we can conclude that while a few variables have been assumed in the creation of our model, overall the result is reasonable and heavily supported with feasible evidence.

In the future, we could investigate the comparison between an ellipse simulation and a parabolic simulation to increase the accuracy of our results. We could also include calculations of the air resistance on the vehicle (despite our efforts to maximise aerodynamics), and calculations of the initial tension which would allow for more definite findings.

Similarly, within our models further research could be done to investigate the material and mass of the launch cup. In our current model we assumed this to be irrelevant, however, more information could be gathered in the future to either confirm or deny this theory. Likewise, we could also investigate the use of different spring materials and the impact of this on the final mass. Based on our findings high carbon steel allowed for the greatest mass to be pulled, however, further calculations may oppose this (for example, if lighter springs were constructed in parallel).

To conclude, our report offers a reasonable solution and explanation for this problem, whilst leaving us with a few variables which could be further explored in the future.