# TEAM 1200

How many satellites can be launched into orbit before astronomers on earth can no longer observe the night sky without interruption?

Abstract

In this report, we considered the coverage of satellites around the earth, and the implications of this on the viewing experience of astronomers around the world. In order to launch sufficient satellites so that we are unable to view the night sky without interruption, we adopted a strategy to deliberately populate the earth's atmosphere with satellites. Considering a mathematical model and verifying our solution with a stochastic model, we calculated that we needed approximately 694,000 satellites in total to achieve our goal.

Introduction

In the last 30 years, the satellite has become an integral part of our daily life. We use them for everything, from navigation to communication, and with their increased use has come an increased number of satellites launched into space. There are many problems associated with launching a small meteorite into the earth's orbit, one of which is that the pictures of distant planets and unexplored galaxies taken by astronomers around the globe are being blocked by an ever increasing number of satellites. As a result of this, astronomical associations are asking for tighter restrictions on the number, size and design of these satellites.[1] In particular this has become an issue with a recent initiative by SpaceX, the aerospace manufacturer run by entrepreneur Elon Musk, known as StarLink which aims to use a massive network of satellites to provide high speed broadband internet coverage to the entire world. The problem with these satellites is that they are covered with a solar panel surface, which reflects sunlight and produces a white dot in the night sky which can be seen by both satellites and the naked eye.[2] Even with the small number of satellites launched during StarLink's trial run, astronomers have already complained about a large amount of interference by the satellites on their shots of the sky.[3] One image shows the effect these satellites had, producing dull white streaks across the entire frame, even though they were only visible for 5 minutes. Clearly this is a growing issue and so in this paper, we will investigate the minimum number of satellites necessary for one to
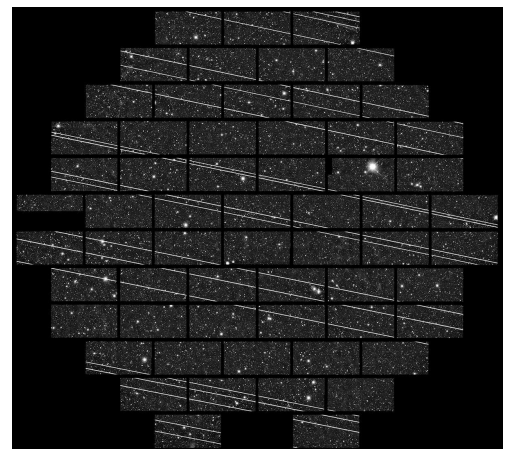
Starlink satellites trail across images from DECam
Cliff Johnson, Clara Martínez-Vázquez, DELVE Survey

[1] Seidler, C. (2017, April 22). Die kosmische Müllkippe[The cosmic dump]. Retrieved from
https://www.spiegel.de/wissenschaft/weltall/weltraumschrott-esa-tagung-in-darmstadt-ohne-ergebnisse-a-1144174.html

[2] Hall, S. (2019, June 01). After SpaceX Starlink Launch, a Fear of Satellites That Outnumber All Visible Stars. Retrieved from
https://www.nytimes.com/2019/06/01/science/starlink-spacex-astronomers.html

[3] Lu, D. (2019, November 19). SpaceX's Starlink satellites are interfering with astronomy again. Retrieved from
https://www.newscientist.com/article/2223962-spacexs-starlink-satellites-are-interfering-with-astronomy-again/

be visible by telescope, regardless of where we are on earth or in which direction we are looking.

Definitions

Defining the Question

We have interpreted the question to be asking us to determine the number of new satellites that must be launched until astronomers can no longer make observations of the night sky without interruption by a satellite. Within the context of this question we have defined "satellites" as objects artificially and intentionally launched into space such that they enter an orbit around Earth. We consider "interruption" to be any perceptible change in the observations of an astronomer using an optical telescope due to the presence of one or more of these artificial satellites. We are also considering an "astronomer" to be any person that uses an optical telescope to observe the night sky.

Defining our Variables

TFOV refers to the True Field of View of a telescope and quantifies the amount of sky visible in a particular scope with a particular eyepiece. For example, if a particular telescope/eyepiece combination provides a 1° TFOV, two stars that are separated by exactly 1° will just fit into the eyepiece field, with each star on opposite edges of the field.[4]

Assumptions

- We do not believe that the time and cost of launch, and the possibility of satellite failure are pertinent to the question, so we have assumed that we have enough capacity and resources on earth to launch all satellites before previously launched satellites deorbit. This should be commercially and practically viable if we launch our satellites into low to medium-earth orbit paths.

- The quality, zoom and most importantly, TFOV of telescopes used by different astronomers will vary drastically. Therefore we have established a theoretical 'average telescope' which we will be using in our modelling. This telescope has a TFOV of 1.02°, which is the mean from a NASA list of telescope eye-pieces available for general purchase.[5] Whilst specialised observatories are likely to have telescopes available with significantly greater magnification factors, and hence significantly smaller TFOVs, we believe it would be unrealistic to consider these cases as the number of satellites necessary would be absurdly large.

- We also assume that we are launching satellites for the specific purpose of interrupting telescope observation, so our satellites will be large and reflective enough to be observed by the aforementioned telescopes. There are currently estimated to be 5774 satellites in orbit[6], however

[4] Thompson, R., & Thompson, B. (n.d.). Astronomy Hacks. Retrieved from
https://www.oreilly.com/library/view/astronomy-hacks/0596100604/ch04s15.html

[5] *Telescope Field of View - How much can you see?* [PDF]. (n.d.). NASA Space Math.

[6] Andy. (2020, May 27). How many satellites are orbiting the Earth in 2020? Retrieved from
https://www.pixalytics.com/satellites-orbiting-earth-2020/

there is no data available on whether these satellites are visible or not by the aforementioned telescopes. Thus, in our modelling, we do not consider existing satellites in orbit.

- Finally, we will not consider the varying exposure times between telescopes when creating a telescopic image of the night sky, this factor is irrelevant as our model ensures that there will always be a satellite present in any image at any time.

Mathematical model

Let us consider the situation below, where all the satellites lie somewhere on a sphere which completely encapsulates the earth.
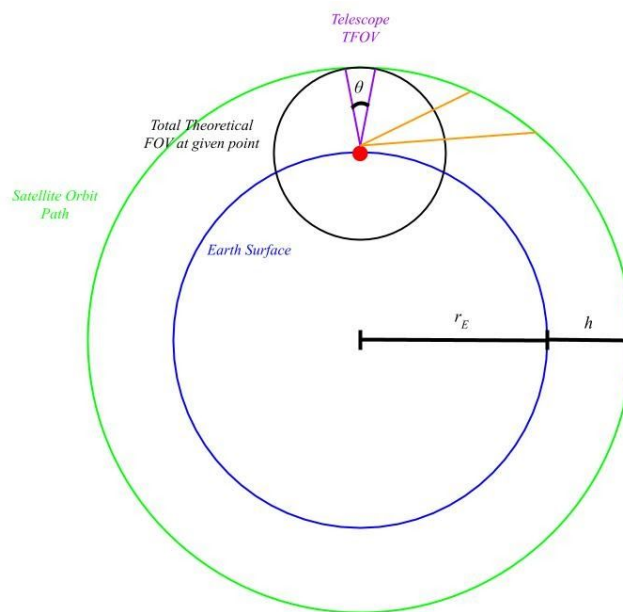
The system with which we have modelled the problem:

r_E is the radius of the earth in metres

h Is the height of the satellite's orbit above the earth's surface in metres

θ Is the angle of the TFOV in radians

Thus, the total spherical surface area on which the satellites may lie is given by:

$$A = 4\pi(r_E + h)^2$$

Assuming that there are no obstacles obscuring our field of view, e.g. trees or buildings, our telescope is able to look around in all directions. We can calculate our solid angle, Ω (in steradians), which gives a 3D representation of our TFOV, θ.

$$\Omega = \theta^2$$

From here, we can find the minimum area of our theoretical sphere which can be viewed by our telescope, A_{TFOV}, as illustrated in the diagram above. Note that this occurs when we are looking straight upwards (given by the purple field of view), as looking in any other direction will result in a larger proportion of our theoretical sphere being encapsulated in the viewing window (as given by the orange field of view).

$$\Omega = \frac{A_{TFOV}}{h^2}$$

$$\therefore A_{TFOV} = \theta^2 h^2$$

As such, the number of satellites we require, N, is given by the total surface area of our theoretical satellite sphere divided by the surface area able to be covered by our telescope.

$$N = \frac{A}{A_{TFOV}}$$

$$N = \frac{4\pi(r_E + h)^2}{\theta^2 h^2}$$

For the purposes of our model, we will be deploying our satellites with a height above earth of 2000 km. This is on the threshold of low and medium earth orbit, which would make the deployment of this satellite relatively commercially viable. We are taking an average value for r_E of 6367.5 km,[7] and a value of theta given by:

$$\theta = \frac{1.02}{180}\pi$$

Thus, at any given instant, our model predicts that we need approximately 694,000 satellites evenly distributed in the sky.

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[7] Imagine the Universe! (n.d.). Retrieved from https://imagine.gsfc.nasa.gov/features/cosmic/earth_info.html

Computational model

We attempted to verify our mathematical estimate using a stochastic model developed in Python.

This model generates a randomized distribution of satellites projected onto a unit sphere. The model is designed to evaluate the amount of empty or "void" spaces within the sky where a telescope may be facing. Hence, an amount of satellites is a valid estimate if the percentage of void spaces is approximately zero (approaching absolute zero is difficult due to the high probability of a small amount of local voids. We consider this is an appropriate approximation as accounting for all voids would render the result inaccurate).

```python
def sample_sphere(n_points):
    """
    Generates a random set of points on a sphere.
    Adapted from: https://stackoverflow.com/a/33977530/5114106
    """
    points = np.random.randn(3, n_points)
    points /= np.linalg.norm(points, axis=0)
    points = zip(points[0], points[1], points[2])
    return list(points)
```

To evaluate the amount of void spaces a k-d tree[8] is used to efficiently query the nearest neighbour of a point in space. Additionally, a 3D convex hull[9] is generated from the satellite points. The "facets" or faces of the convex hull describe a set of triangles of which the centroids represent possible points for a telescope to point to. If the Euclidean distance between the nearest satellite point of a centroid and the centroid itself is greater than the diameter of the projected telescope ray on the orbit sphere then there is a high probability that a ray can be projected at the midpoint between the centroid and neighbour point with no satellites in the projection. That is, a void space would be present.
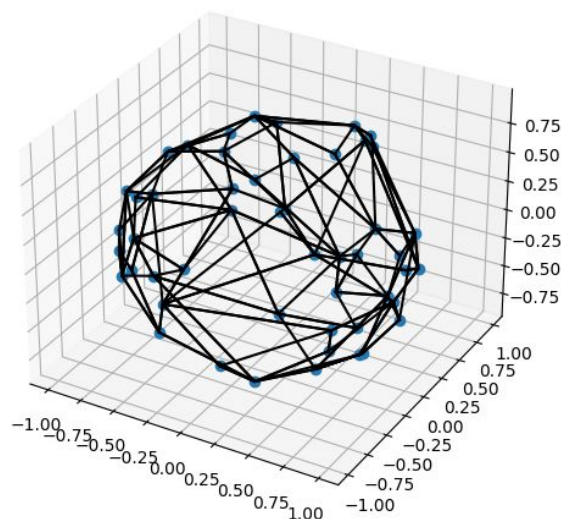
Visualization of randomised satellites (n = 50, blue dots)

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[8] https://en.wikipedia.org/wiki/K-d_tree

[9] https://en.wikipedia.org/wiki/Convex_hull

```python
def evaluate(points):
    """
    Evaluates the percentage of voids for a
    set of satellite points.
    """
    tree = KDTree(points)
    hull = ConvexHull(points)
    print(f"Hull: {len(hull.simplices)} faces")

    centres = []
    for simplex in hull.simplices:
        centres.append(np.mean([
            points[simplex[0]],
            points[simplex[1]],
            points[simplex[2]],
        ], axis=0))
    print("Computed face centroids")

    pool = Pool(processes=processes)
    blocks = np.array_split(centres, processes)
    arguments = zip([tree] * processes, blocks)
    n_void = sum(pool.map(compute, arguments))
    return n_void / len(hull.simplices)
```

The time of the computation increases as the number of satellite points increases (and consequently the number of convex hull facets and centroids). To make the computation tractable the evaluation is split into four separate blocks of centroids, each of which can be processed by a different processor core.

```python
def compute(arguments):
    """
    Evaluates the percentage of voids for a
    block set of centroids.
    """
    tree, block = arguments
    qs = tree.query(block, distance_upper_bound=diameter)
    n_voids = np.count_nonzero(np.isinf(qs))
    print(f"Block complete")
    return n_voids
```

Note that the diameter of the projected ray increases in proportion to the radius of a satellite orbit as it is only dependent on the angle of the field of view of a telescope. Consequently, using a unit sphere rather than a correctly scaled sphere (to the size of a given orbit) is a reasonable approximation.
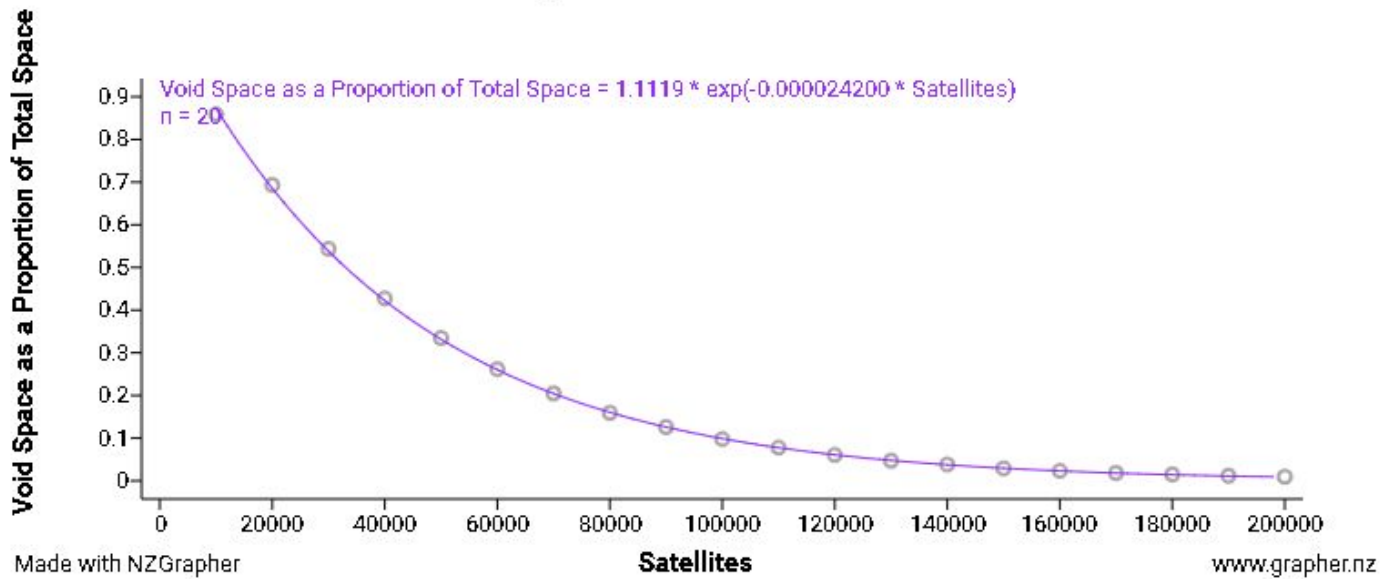
```python
import math
import numpy as np
from multiprocessing import Pool
from scipy.spatial import ConvexHull, KDTree

degrees_view = 1.02
diameter = (math.pi / 360.0) * degrees_view
processes = 4
```

To validate the stochastic model and ensure the results are as expected, a graph is plotted with the void space on the y-axis and the number of randomly distributed satellites on the x-axis.

Void Space over Satellite Count

Void Space as a Proportion of Total Space = 1.1119 * exp(-0.000024200 * Satellites)
n = 20

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The void space proportion decreases rapidly as the amount of satellites increases and then tails off as the proportion approaches zero. This aligns with the expectations of the stochastic model. For our value of 694,000 satellites, we calculated a void space proportion of 5.65×10^{-8} which means that our mathematical estimate is likely to be an appropriate solution to the problem.

Conclusion

After verifying the results of our mathematical model using our computational model we have determined the number of satellites that need to be launched is approximately 694,000. According to a 2020 estimate there are approximately 5774 satellites currently in orbit with only 137 in a medium earth orbit.[10] Therefore the addition of these satellites would represent a 12,019% increase in the total number of satellites and a 506,569% increase in the number of objects in mid earth orbit. In reality with current technology, this may not necessarily be feasible.

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[10] Andy. (2020, May 27). How many satellites are orbiting the Earth in 2020? Retrieved from
https://www.pixalytics.com/satellites-orbiting-earth-2020/

Discussion

There are some possible confounding variables that may change the necessary number of satellites to be launched in a real life situation.

Satellite collisions

With an estimated value of 694,000 satellites at 2000 kilometres above the Earth's surface, the average area around a uniformly distributed satellite is approximately 1.27×10^9 square meters. As our satellites are in a low-mid orbit, they would have thrusters capable of moving to avoid the trajectories of other satellites when performed in advance of the collision. Hence, the probability of an unavoidable satellite collision is low.

Satellite formation

In making these calculations we have made the assumption that each satellite has either a fixed or random position around the earth in low-mid earth orbit and although having satellites stationary in space would not be possible, we are able to create a stationary pattern of satellites in the sky by incorporating a formation known as the Walker constellation. In this arrangement each satellite is positioned so that as it travels around the earth its relative position to adjacent satellites remains at a fairly constant distance, meaning the distribution of satellites in the sky remains constant.[11] This kind of constellation was used in the StarLink project to allow the entire earth to have equal coverage from satellites in space.[12]

Starlink Initial Phase

1,584 satellites into 72 orbital planes
of 22 satellites each

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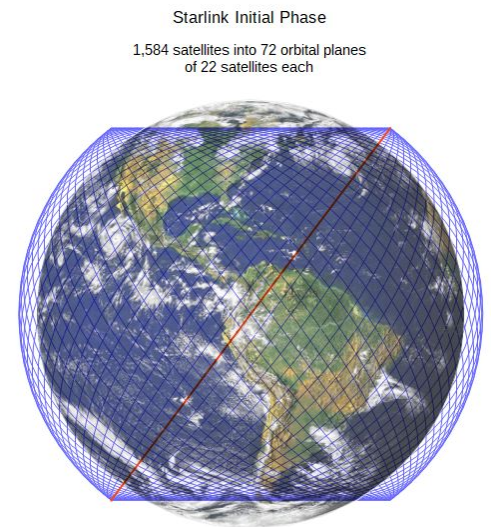
[11] Satellite constellation. (2020, July 12). Retrieved from https://en.wikipedia.org/wiki/Satellite_constellation
[12] Starlink. (n.d.). Retrieved from https://en.wikipedia.org/wiki/Starlink
Image sourced from above citation