Generalised Inverse Limits

Yuki Maehara

Supervisor: Sina Greenwood

Given a sequence $\boldsymbol{X} = X_0, X_1, \ldots$ of topological spaces and a sequence $\boldsymbol{f} = f_0, f_1, \ldots$ of mappings such that $f_i : X_{i+1} \to X_i$ for each $i \in \mathbb{N}$, the *inverse limit* of this pair of sequences (called an *inverse sequence*) is the space defined by

$$\varprojlim \langle \boldsymbol{X}, \boldsymbol{f} \rangle = \left\{ (x_i)_{i \in \mathbb{N}} \in \prod_{i \in \mathbb{N}} X_i : \forall i \in \mathbb{N}\ x_i = f_i(x_{i+1}) \right\}.$$

Inverse limits have been studied for more than 50 years. One of the interesting things about them is that you can start with really simple factor spaces and bonding functions and obtain a very complicated inverse limit. An example of a class of such complicated spaces is the class of indecomposable continua. A *continuum* is a non-empty compact connected metric space. A continuum is *indecomposable* if it cannot be written as the union of two of its proper subcontinua. There is a well-known theorem that gives a sufficient condition for indecomposability of inverse limits.

Recently, a generalisation of inverse limits was introduced, where bonding functions are allowed to be set-valued. One natural question to ask is: how can we generalise the theorem on indecomposability? Ingram[1], Varagona[3], Kelly and Meddaugh[2] and other people have generalised it in several ways, and many results have been published. However, all of these results rely heavily on so-called the *full projection property*. A connected inverse limit $\varprojlim \langle \boldsymbol{X}, \boldsymbol{f} \rangle$ is said to satisfy the full projection property provided for each proper subcontinuum C of $\varprojlim \langle \boldsymbol{X}, \boldsymbol{f} \rangle$, $\pi_n(C) \neq \pi_n(\varprojlim \langle \boldsymbol{X}, \boldsymbol{f} \rangle)$ for all but finitely many n. Every regular inverse limit has this property, but it is not the case for generalised inverse limits. In this talk, we investigate the necessity of the full projection property for indecomposability of generalised inverse limits.

References

[1] W. T. Ingram. Inverse limits with upper semi-continuous bonding functions: problems and some partial solutions. *Topology Proc.*, 36:353–373, 2010.

[2] James P. Kelly and Jonathan Meddaugh. Indecomposability in inverse limits with set-valued functions. *Topology Appl.*, 160(13):1720–1731, 2013.

[3] Scott Varagona. Inverse limits with upper semi-continuous bonding functions and indecomposability. *Houston J. Math.*, 37(3):1017–1034, 2011.