BLENDERS HUNTING: THE SEARCH FOR WILD CHAOS

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1. BACKGROUND

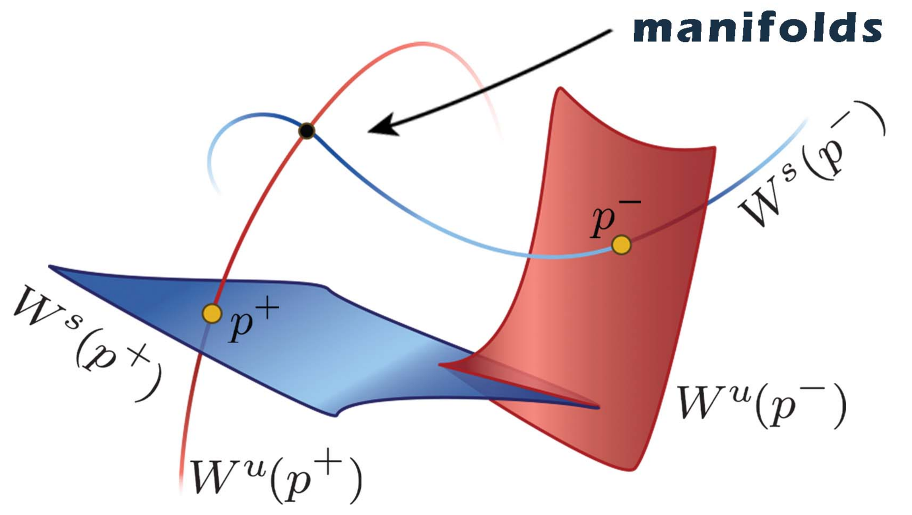
- **Wild chaos** is a new type of chaotic dynamics that could model robust chaos in **nature**.

ROBUST CHAOTIC DYNAMICS: CHAOS THAT PERSISTS UNDER PERTURBATIONS

- Chaos can arise from intersections between curves. Unlike classical chaos, in wild chaos those intersections do not disappear.

CLASSICAL CHAOS

Non-robust int. between manifolds



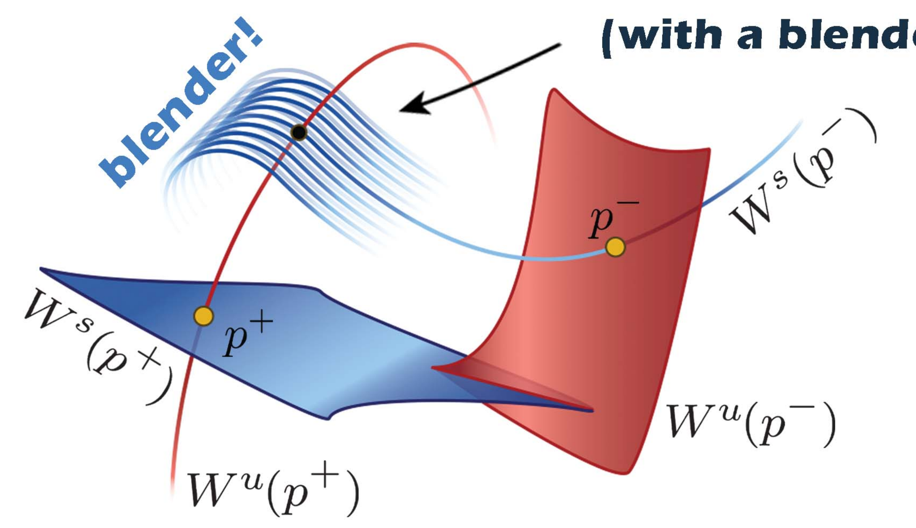
- Wild chaos can be generated by a geometric structure called a **blender**. Blenders behave as higher-dimensional objects.

HOW DO WE KNOW THERE IS A BLENDER? WHEN A 1D CURVE LOOKS LIKE A SURFACE

- To find a blender, we developed **advanced numerical techniques** to compute extremely long curves called **manifolds**.

WILD CHAOS

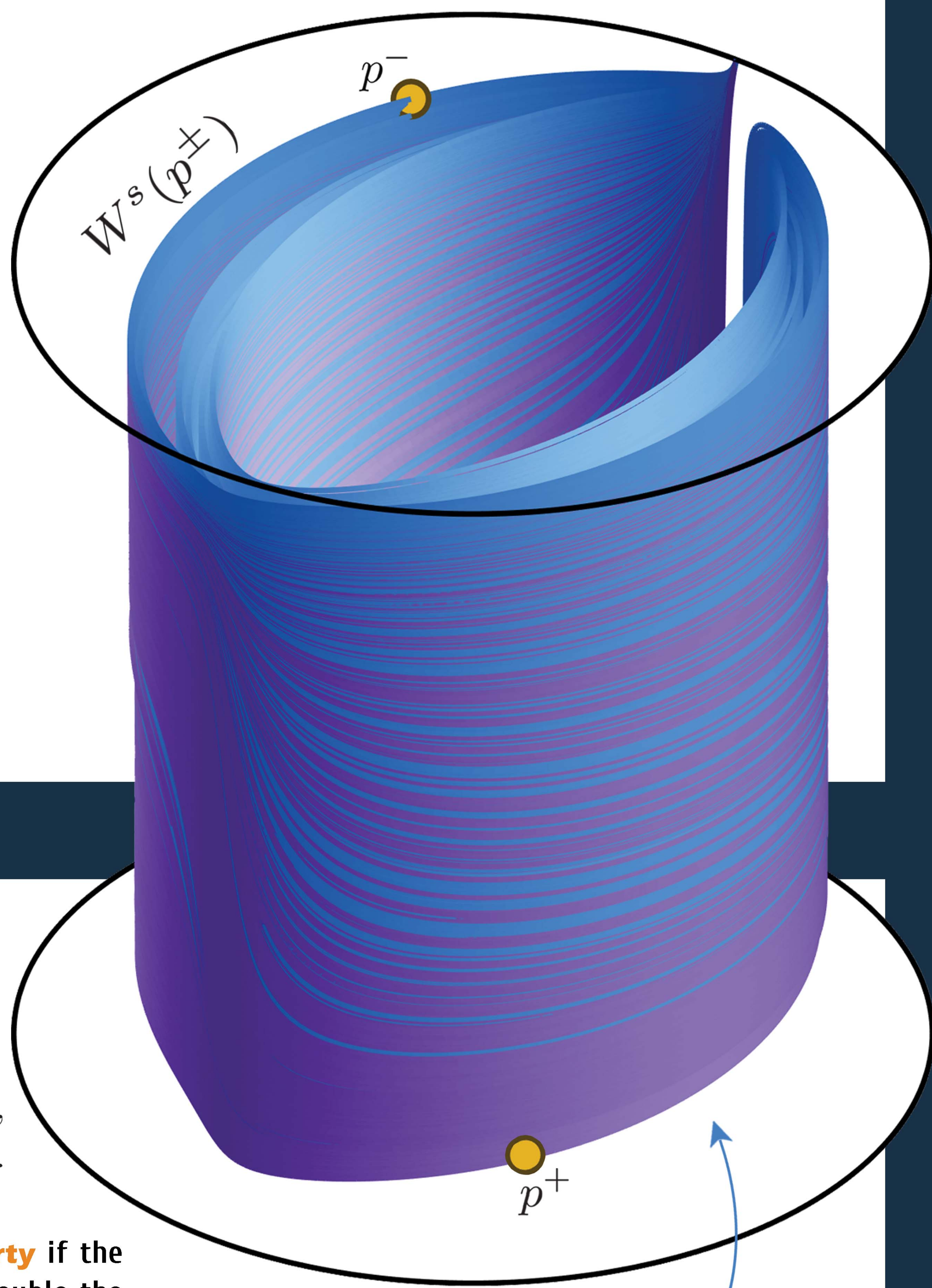
Robust int. between manifolds (with a blender)



A BLENDER IN A 3D HÉNON-LIKE MAP

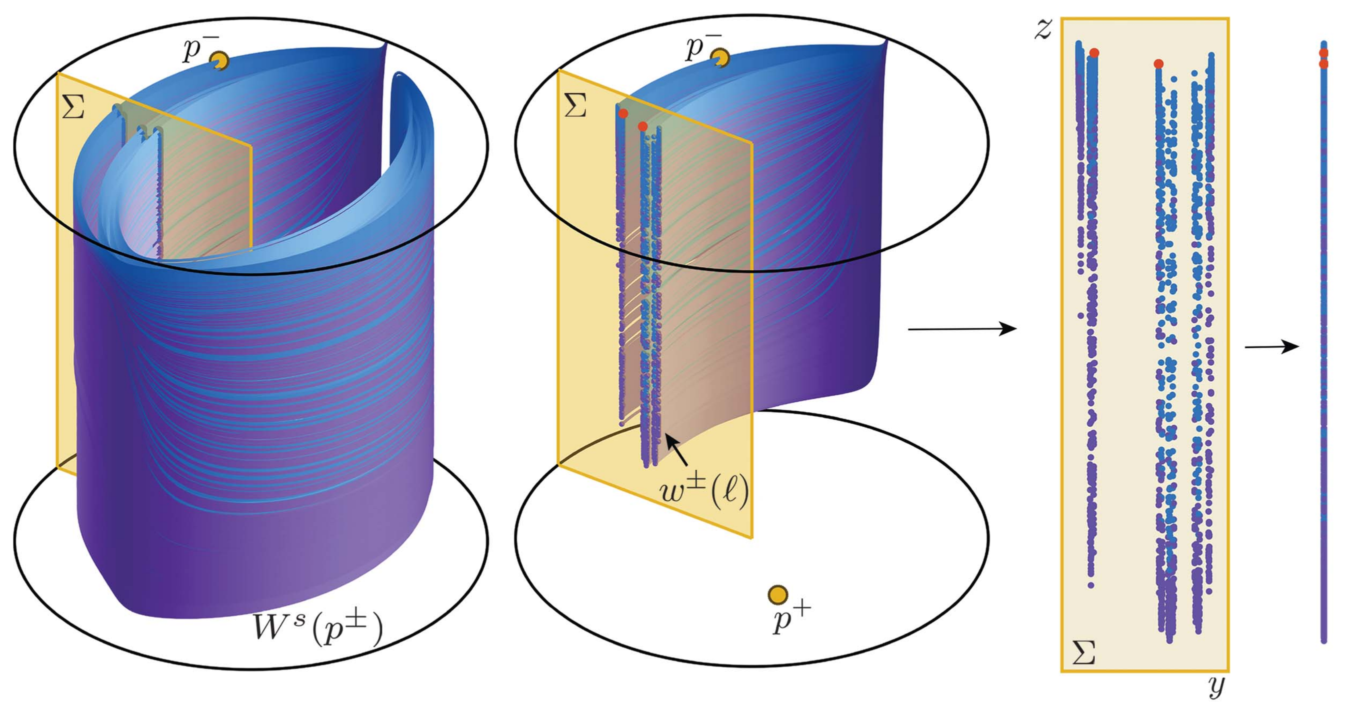
$$H(x,\ y,\ z) = (y,\ a - y^2 - b\,x,\ \xi\, z + y)$$

They are just two curves!

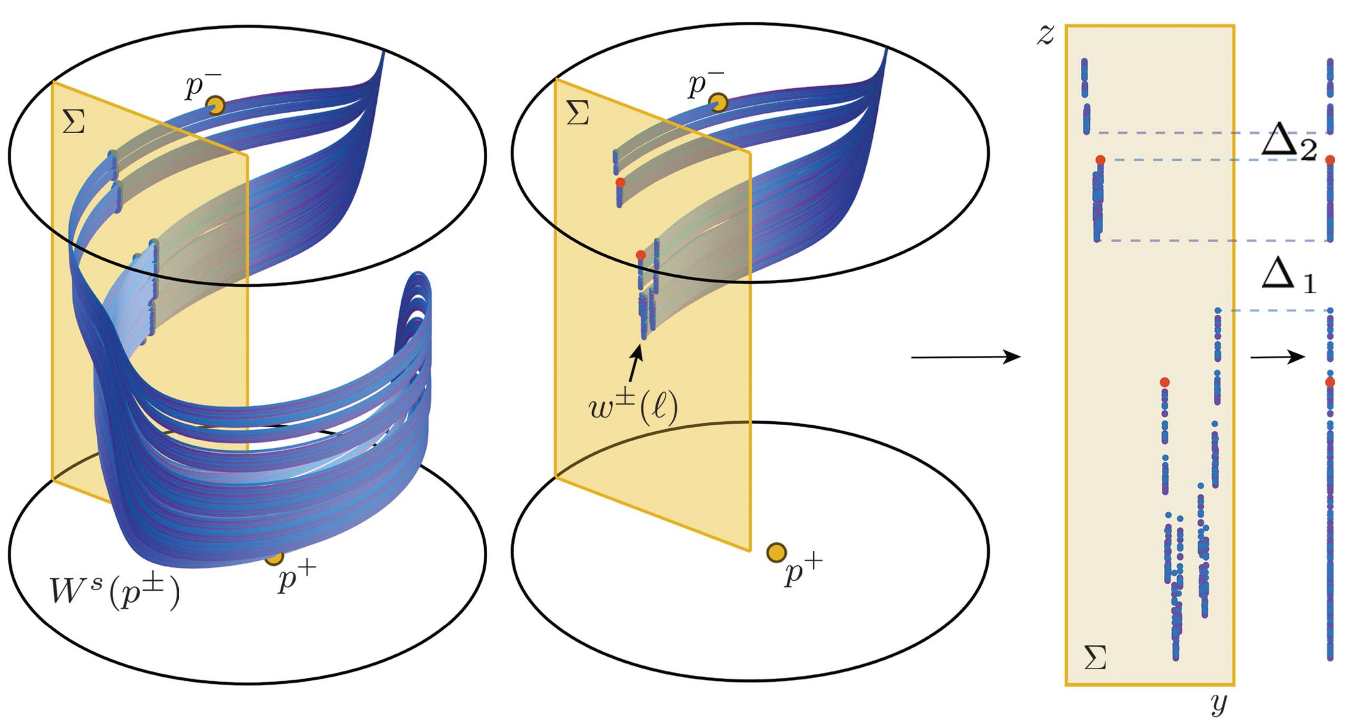


2. METHOD: WHEN IS THERE A BLENDER? THE CARPET PROPERTY

CASE 1: blender for $\alpha = 4.2$, $\beta = 0.3$, $\xi = 1.2$



CASE 2: no blender for $\alpha = 4.2$, $\beta = 0.3$, $\xi = 1.8$



- We **intersect** the manifolds with a plane Σ and consider the sets of intersection points:

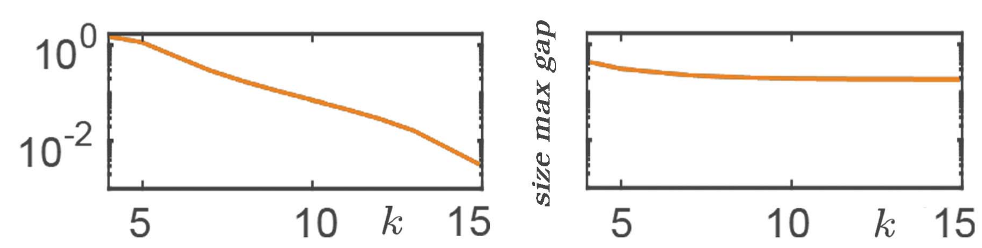
$$\{w^-(\ell)\} := \{(x,y,z) \in W^s(p^-) \cap \Sigma\},$$
$$\{w^+(\ell)\} := \{(x,y,z) \in W^s(p^+) \cap \Sigma\}.$$

- A manifold has the **carpet property** if the **maximum gap** goes to zero as we double the amount of ($\ell = 2^k$) intersection points $\{w^\pm(\ell)\}$.

WHEN THE MANIFOLDS HAVE THE CARPET PROPERTY, THEN THE SYSTEM EXHIBITS A BLENDER

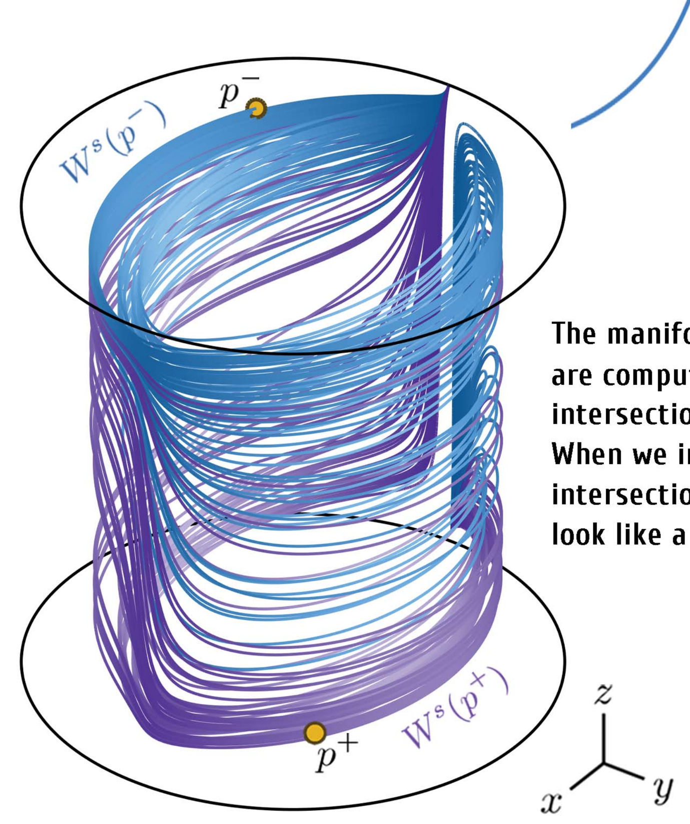
CONVERGENCE OF MAXIMUM GAP

CASE 1: blender | **CASE 2: no blender**



The maximum gap converges to zero with constant slope.

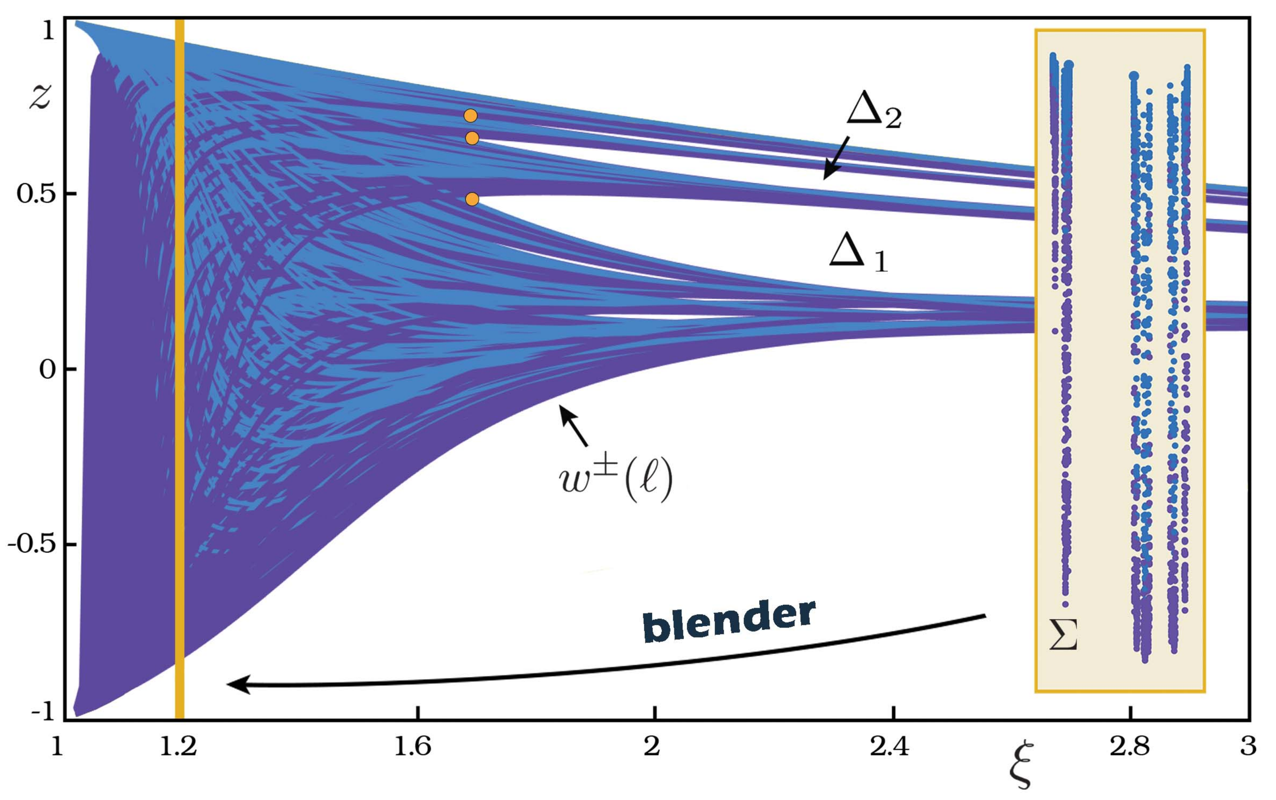
The maximum gap converges to a non-zero value.



The manifolds $W^s(p^\pm)$ are computed here up to 2^7 intersection points with Σ. When we increase the intersections, they start to look like a surface.

3. RESULTS: CARPET PROPERTY AS A FUNCTION OF ξ

- The generation and desappearence of blenders can be studied as a **function** of ξ.



- We find a **recurrent pattern** in how the intersection points $w^\pm(\ell)$ reorganise and the gaps Δ_k close as ξ decreases.

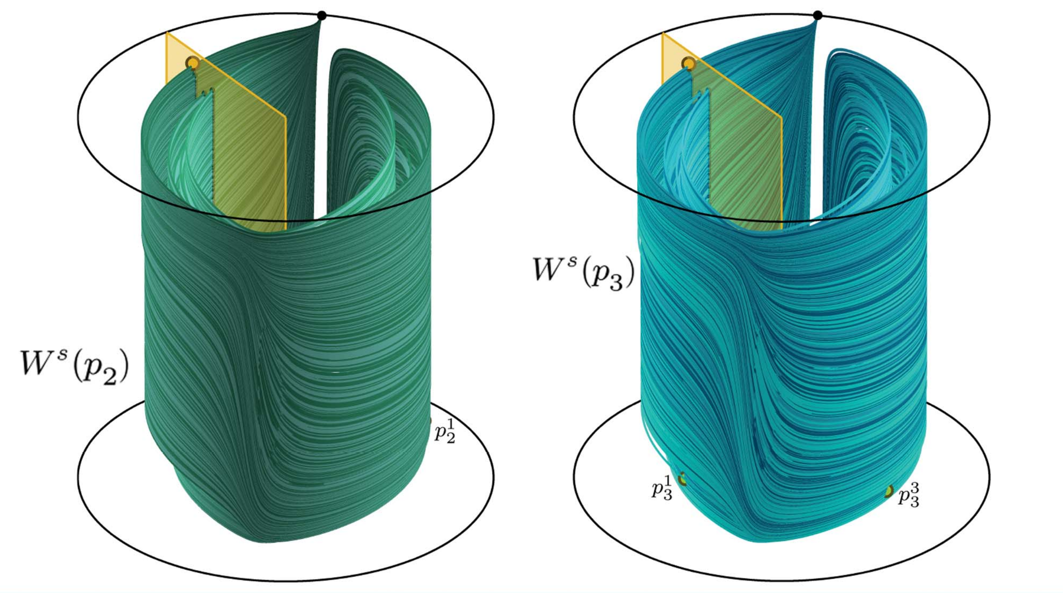
WE CAN PREDICT AND DETERMINE EXACTLY WHICH PART OF THE MANIFOLD IS RESPONSIBLE FOR EACH GAP Δ_k.

- Every gap Δ_k closes **sequencially**; the size δ_k of Δ_k is zero just before δ_{k+1}.
- We **accurately estimate** ξ^* such that there is a blender for all $1 < \xi < \xi^*$.

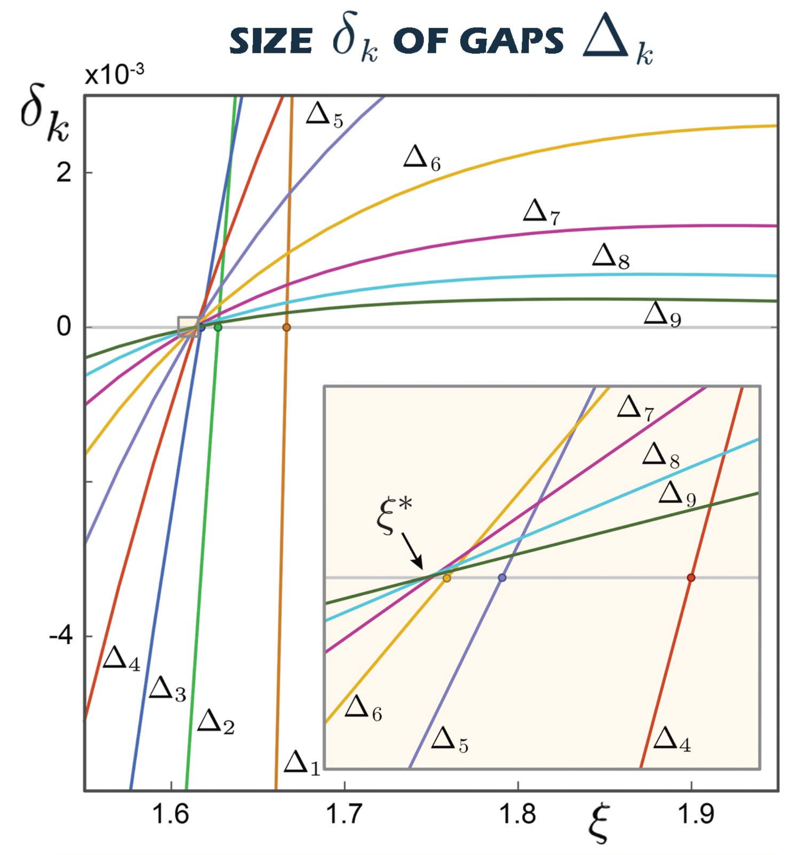
IT IS EFFICIENT TO CHECK THE LIMIT ξ^* FOR WHICH THE SYSTEM HAS A BLENDER

ON GOING WORK:

- There is a similar pattern for when the parameter β is negative.
- Things change when $\xi < -1$; we already have evidence that other manifolds are responsible for the gaps.



SIZE δ_k OF GAPS Δ_k



Blender for $\xi < \xi^* \approx 1.6135\cdots$

[1] Wild chaos and blender theory: C. Bonatti and L.J. Díaz (1996).
[2] Wild chaos and blender theory: C. Bonatti, L.J. Díaz and M. Viana (2005).
[3] Numerical techniques: B. Krauskopf and H.M. Osinga (1998).
[4] Carpet property: S. Hittmeyer, B. Krauskopf, H.M. Osinga and K. Shinohara (2018) and (2020).