INTO THE WILD: A JOURNEY TO CHAOS IN FOUR DIMENSIONS

Juan Patiño-Echeverría
Prof. Hinke Osinga
Prof. Bernd Krauskopf

University of Auckland, Waipapa Taumata Rau, New Zealand
Department of Mathematics

THE UNPREDICTABLE NATURE OF CHAOS

Why do we often need a raincoat when the forecast promises sunshine? Why do some mathematical models seem to predict the future better than others?

The meteorologist Edward Lorenz found that the problem was unpredictability; to explain it, he introduced the concept of ***chaos***.

WHAT DO WE DO?

Different forms of chaos exist in four dimensions. We aim to understand what geometric ingredients create such chaos. In particular, we study the following system of equations

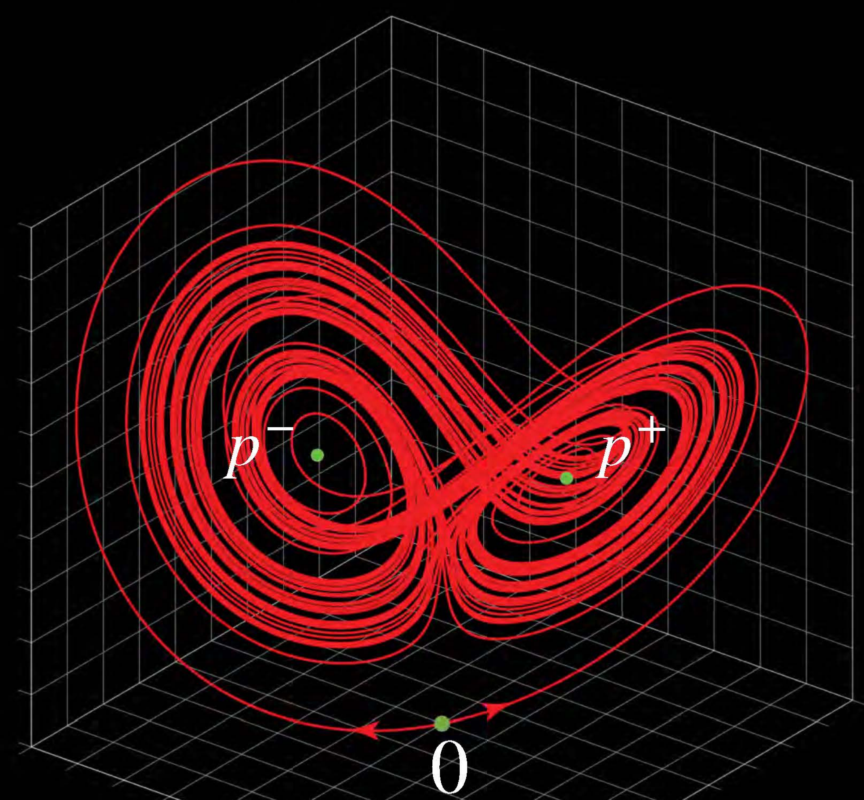
$$\begin{cases} \dot{x} = \sigma(y - x), \\ \dot{y} = \rho x - xz - y, \\ \dot{z} = xy - \beta z + \mu w, \\ \dot{w} = \quad -\mu z - \beta w. \end{cases}$$

HOW?

We use ***bifurcation theory***, which identifies dramatic changes in behaviour as parameters vary; here, we study ρ and μ.

Our approach is to construct a ***kneading diagram [1]*** in the (ρ, μ)-plane that helps us to visualise where the different types of chaos exist.

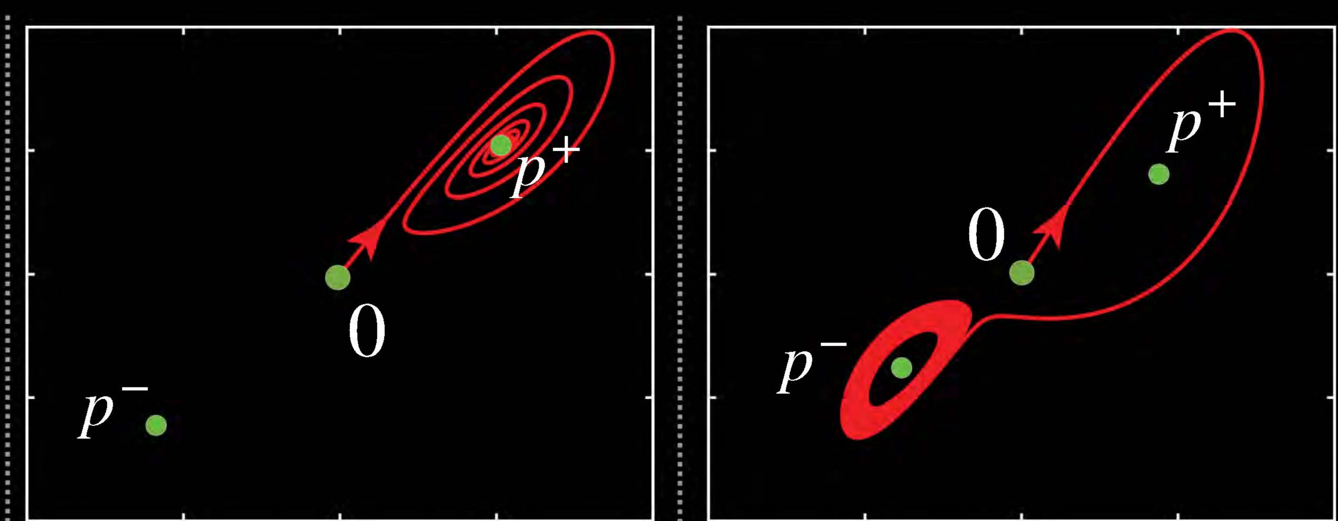
1 We compute a special curve, called the *unstable manifold of the origin* ($W^u(0)$), for many different parameters in the (ρ, μ)-plane.



3D projection of $W^u(0)$ when $\rho = 100$, $\mu = 1.75$, $\beta = 8/3$, $\sigma = 10$. Observe how $W^u(0)$ oscillates around co-existing equilibria p^+ and p^-.

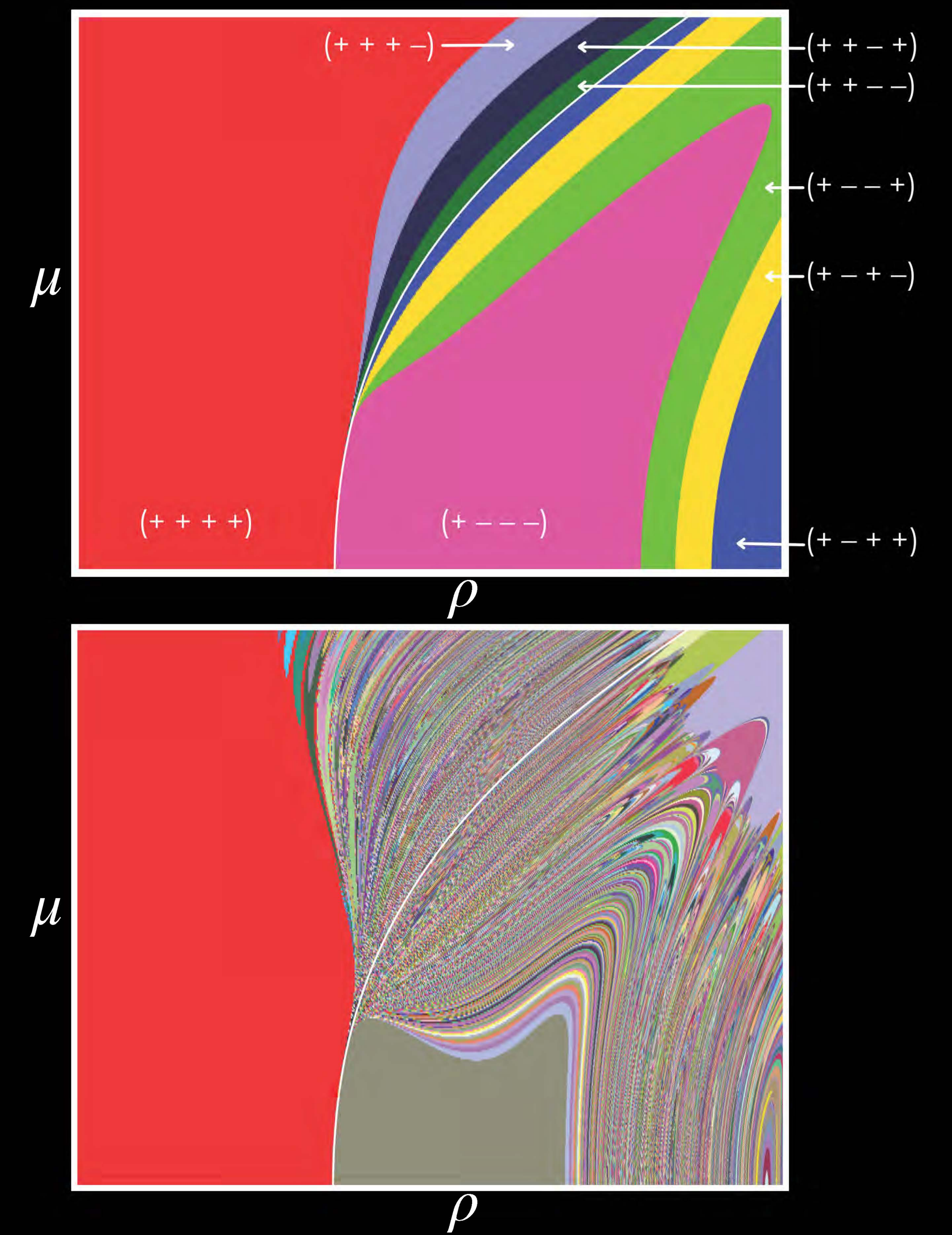
2 Next, following from the origin, we assign a sequence of + and – depending on whether $W^u(0)$ goes around the equilibrium points p^+or p^-, respectively. For example

View from (x, y)-plane



Sequence (+ + + + ...) (+ – – – ...)

3 Because of symmetry, we only need to consider sequences that start with +. We assign colours to distinguish sequences. For example, up to the first four, respectively twelve symbols, the (ρ, μ)-plane is coloured like this



Results: • Every change in colour tells us that the system is changing drastically. With longer sequences, the kneading diagram becomes complex; in some areas, the colours change instantly in any direction; this gives us a clue of where different types of chaos could exist. • Our research has enhanced our understanding of how new forms of chaos arise in vector fields.

References

[1] Barrio R, Shilnikov A, Shilnikov L 2012 Kneadings, Symbolic Dynamics and Painting Lorenz Chaos *Int. J. Bifur. Chaos Appl. Sci.*