TEAM #1048

The Question Statement:

How many satellites can be launched into orbit before astronomers on earth can no longer observe the night sky without interruption?

Summary

We have decided to call the release point of satellites a burst, thus 1 burst would release certain amount of satellites. We have assumed that 1 burst releases 4 satellites in directions such that the path of each one is perpendicular to the path of each of its immediate neighbours. We then used a sample photo taken by our specified telescope, the Victor M. Blanco telescope, to calculate how many photos we can fit into the shell of the the low-Earth orbit, the area of a satellite, the area of the light the satellite reflects and then the average area of the line a satellite makes through the photo with an area we discovered previously. This line is the area of the pathway in the sky which each satellite makes. Using this area of a line, we could determine how many lines could pass through a photo before 3% of the photo was completely obscured which we defined to be an interruption. The number of satellites in a photo is 2 times the number of lines, because of the definition of the burst releasing 4 satellites. 2 satellites that are being released in the opposite direction will form a complete line. By multiplying the number of satellites in a photo to the number of photos we can fit into the shell of the low-Earth orbit we can determine the maximum number of satellites we can launch into the orbit before every photo we take of the night sky has over 3% of it obscured by lines from passing satellites, meaning all observations of the night sky will be interrupted. The final number of satellites we found we could launch into low-earth orbit before this interruption was approximately 45,600 satellites

1 Introduction

Launching more and more satellites is providing internet access to the most remote places on earth and improving the lives of those who lack such access. However, the brightness of these satellites are causing widespread concern amongst the astronomy community. With thousands of objects orbiting overhead, astronomers already struggle to gather data. Our goal is to deduce how many satellites can be sent into the Earth's orbit before astronomers can no longer create long exposure photos of the night sky without obstruction.

1.1 Definition of Question

Satellites will be defined as internet satellites which are artificial satellites orbiting in low-Earth orbit which facilitate low-latency broadband internet service to earth. We have defined this question to only concern these internet providing satellites because these are the satellites that are being launched into orbit in high quantities to facilitate internet access to remote areas. The satellite we will be basing our model off is the SpaceX Starlink satellites as SpaceX plans to launch over 12,000 of these into orbit.
Orbit will be defined as the low-Earth orbit as this is the orbit in which internet satellites orbit. The low-Earth orbit is an orbit that is relatively close to Earth's surface. At an altitude between 1000 km and 160 km above Earth.
Observing the night sky is defined to be using large ground-based telescopes to collect light for minutes or sometimes hours at a time which creates long exposure photographs of the night sky.
Interruption is defined as a satellite passing through the frame of the photo, leaving a bright streak through the photograph. We consider the point at which the data from the photo can no longer be used for scientific reasons as the point at which the obstruction is over 3% of the photo.

1.2 Assumptions

1. Assume the earth to be a perfect sphere
2. Orbit is circular
3. Satellites are orbiting in the low-Earth orbit at an altitude of 550km
4. Observation of the night sky is by a long exposure photograph taken by the Victor M. Blanco Telescope.

We assume the earth to be a perfect sphere as it is a good enough estimate of the earths shape and will let us do calculations for dimensions of the earth using simpler equations.
The orbit will be assumed to be circular because this will mean that the satellite is the same altitude around the spherical earth at every point in its path.
The reason we made the assumption that the altitude of satellites is 550km is because the Starlink satellites, which we have used to base our definition of a satellite, all orbit at 550km.
We chose to assume that observations were being taken from the Victor M. Blanco telescope because the photos we found of the night sky with streaks from satellites. This means this is a telescope used for standard optical astronomy at it is also sensitive to the effects of satellites passing through the frame.

2 Approaches to Question

Our approach has been divided into the subsections. When the satellites are launched into the space, initially they are packed into a pile and then at a certain point in the space they are released. We have decided to call this a burst, thus 1 burst would release certain amount of satellites. We have assumed that 1 burst releases 4 satellites in directions such that the path of each one is perpendicular to the path of each of its immediate neighbours. This is shown in the diagram below. We then calculated the areas of a photo taken by our specified telescope, how many photos we can fit into the shell of the the low-Earth orbit, the area of a satellite, the area of the light the satellite reflects and then the average area of the line a satellite makes through the photo with an area we discovered previously. This line is the area of the pathway in the sky which each satellite makes. Using this area of a line, we could determine how many lines could pass through a photo before 3% of the photo was completely obscured which we defined to be an interruption. The number of satellites in a photo is 2× number of lines, because of the definition of the burst releasing 4 satellites. 2 satellites that are being released in the opposite direction will form a complete line. By multiplying the number of satellites in a photo to the number of photos we can fit into the shell of the low-Earth orbit we can determine the maximum number of satellites we can launch into the orbit before every photo we take of the night sky has over 3% of it obscured by lines from passing satellites, meaning all observations of the night sky will be interrupted.

2.1 Burst

We define a "burst" to be a release of 4 satellites radiating outwards from a given point, and these bursts are evenly distributed across the surface of the Earth. These bursts are the starting

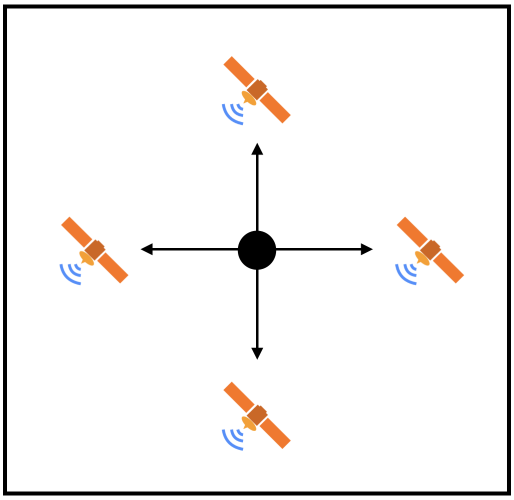

Figure 1: Diagram of satellites radiating outwards from a burst

and ending points of each line which appears in photos of the night sky. These bursts are useful because we can use them to determine the number of satellites in a given photo because we simply have to calculate how many lines can be created by a satellite moving through the frame before the obstruction exceeds 3% and then the number of lines can be multiplied by 2 to find the number of satellites in the photo since 2 satellites radiate from opposite points of a burst and form a line through a photo.

2.2 Area Approximations

The area of the photo can be calculated as the following(The given values are taken from the website that we have cited below):

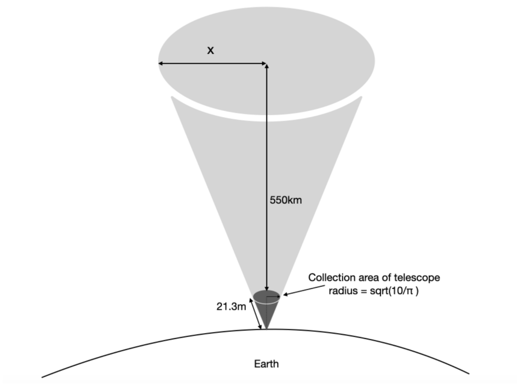

Figure 2: Diagram showing the size of the photograph - black coloured part is the telescope

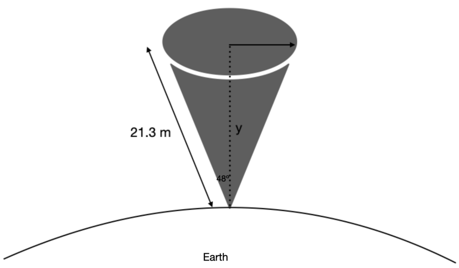

Figure 3: This is just the black part of the previous diagram maximised

$$\arcsin \frac{\sqrt{\frac{10}{\pi}}}{21.3} = 4.8^\circ \tag{2.1}$$

$$y = \frac{\sqrt{\frac{10}{\pi}}}{\tan 4.8^\circ} \tag{2.2}$$

$$x = \tan 4.8^\circ \times (y + 550) = \tan 4.8^\circ \times (\frac{\sqrt{\frac{10}{\pi}}}{\tan 4.8^\circ} + 550) = 47.9689... \approx 48 \tag{2.3}$$

Thus from the above, we know the area of the photo, taken by our specified telescope, the Victor M. Blanco telescope is (A_p)

$$A_p = \pi \cdot (48km)^2 = 7238.229474km^2 \approx 7238km^2 \tag{2.4}$$

We then calculate the area of the shell of the low-Earth orbit (altitude=550km) using the equation for the surface area of a sphere.

$$A_S = 4 \cdot \pi \cdot (6371km + 550km)^2 = 6.02 \cdot 10^8 km^2 \tag{2.5}$$

The area of the shell is the area where all the satellites will be orbiting and therefore, the more satellites we launch the fuller this shell will become.
Let the number of photos we can fit into the shell of the low-Earth orbit be N_p

$$N_p = A_S/A_p = 6.02 \cdot 10^8 km^2/7238km^2 \approx 83160 \tag{2.6}$$

We've divided the area of the shell by the area of the photos to see how many photos will need to be taken to observe the night sky. It'll also average the number of satellites across the number of photos so that we can have no greater than 3% obstruction in any of these observations of the night sky
The area of the base of the satellites is A_{SB}

$$A_{SB} = (140 \cdot 10^{-5} km) \cdot (60 \cdot 10^{-5} km) = 8.4 \cdot 10^{-7} km^2 \tag{2.7}$$

This area is the area of the satellite and we found this area based off the measurements of the SpaceX Starlink satellite which we researched and based our model on. Sources were quite vague on the measurements of this satellite. However, most agreed it was about the size of a table so we got this measurement by measuring a table at school and finding the area of the satellite.

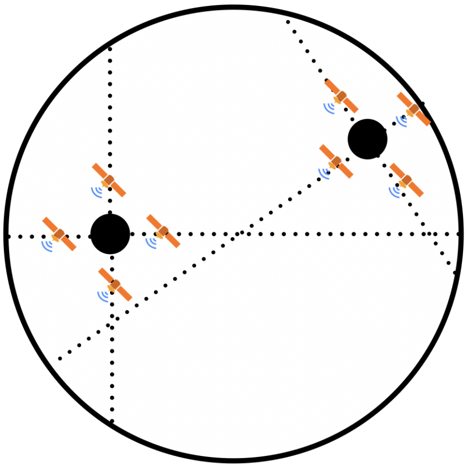

Figure 4: Diagram of the pathways of satellites radiating outwards from two bursts in a photograph

Let the satellites release such that the path of each one is perpendicular to the path of each of its immediate neighbours. Then we can say that each burst released in a photograph frame will form two arbitrary chords.

Claim: The average height of a circle is $\frac{\pi r}{2}$.
Proof: We want the height to be such that multiplying it by the diameter of the circle, we get the total circle area by the mean value theorem. Thus, letting the average height be h_{av}, we get

$$h_{av} \cdot 2r = \pi r^2.$$

This implies that

$$h_{av} = \frac{\pi r}{2}.$$

This proves our claim.

From this, we find that the average length of each chord is

$$\ell_{av} = \frac{\pi r}{2} = \frac{\pi \cdot 46.2km}{2} = 72.6km.$$

Also, we know that the light the satellite reflects from sunlight will make the satellite appear with a width of 10.1 km. This is proven in section "2.3 Visibility of Satellite". Let A_{SP} denote the average area of the pathway in the sky which each satellite makes. Thus,

$$A_{SP} = (10.1km) \cdot (72.6km) = 733km^2.$$

2.3 Visibility of Satellite

We found the size of the light reflected from the satellite in proportion to the size of the photo by using the aspect ratio of a long exposure photograph taken by the Victor M. Blanco telescope. The whole image was 1200x1086 pixels with the average width of each streak of Starlink's satellites being $\frac{1}{662}$ of the photo or 1.8 pixels. As the total size of the photo is $6705.5km^2$, we can now approximate the ratio of pixel size to real size of photograph which we found to be 1px : $5.59km^2$. Thus, the true size of the light reflected off each satellite is determined to be 1.8 $\cdot$5.59 =10.1km^2.

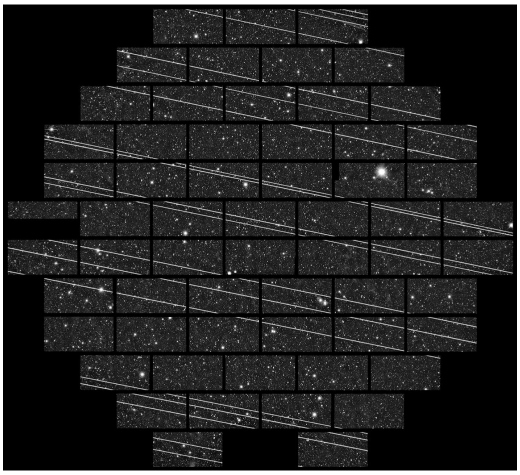

Figure 5: Long exposure photo taken by Victor M. Blanco telescope.

This photo was used to approximate the size of the light reflected by each satellite

2.4 Estimation of Answer

To find an answer for how many satellites we can launch into the low-Earth orbit before astronomers on earth can no longer observe the night sky without interruption, we will look at a single photo taken by our selected telescope. IF 3% of this photo is obstructed by lines from satellites, it has been interrupted. 3% of this photo is

$$3\% = 0.03 \times Area = 0.03 \times 7238.229 = 201.1662km^2 \tag{2.8}$$

Let the number of lines in the photo for there to be 3% obstruction be denoted as x

$$x = \frac{201.1662}{10.1 \times 72.5707903} \tag{2.9}$$

The number of satellites in a photo is equal to $2x$ because a line is composed of 2 satellites as defined in the burst.

$$2x = 2 \times \frac{201.1662}{10.1 \times 72.5707903} = 0.5489108073 \tag{2.10}$$

Finally, we can calculate the total number of satellites we can launch into orbit by multiplying the total number of satellites in a photo by the total number of photos we can fit into the shell of the low-Earth orbit

$$N_p \times 2x = 83160.14062 \times 2x = 83160.14062 \times 0.5489108073 = 45647.49992... \approx 45,600 \tag{2.11}$$

Therefore the answer is that you can launch an estimated $\boxed{45,600}$ satellites into the earths lower orbit before every photo has at least 3% obstruction and an astronomer cannot observe the night sky without interruption

3 Limitations/Uncertainties

- Satellites that are covered in the shade of the Earth would not be visible as they do not reflect any light.
- We did not take into account of the lines that overlap each other. As this would be minimal compared to the whole surface area of the shell we have excluded this calculation. However, in real life this would affect the number of satellites.
- There would be uncertainties coming from the assumptions that we made with the key terms of "orbit", "astronomer" and "without interruption." The orbit range stays constant.

Conclusion

Based on our calculations using the average area of a line passing through a photo of defined area taken by our specified telescope, we deduced the number of lines passing through each photo and then deduced the number of satellites in each photo. Multiplying this to the number of photos in the low earth orbit shell gave us an answer that approximately 45,600 satellites could be launched before serious interruption to astronomers interrupted all observations of the night sky.
currently there are around 2000 satellites in orbit in the low-earth orbit and the new project from SpaceX, Starlink is planning to send up 12,000. These will not interrupt astronomic observations as we have shown.

References

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Section 1.2 https://www.vox.com/science-and-health/2020/1/7/21003272/space-x-starlink-astronomy-light-pollution

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Section 2.1 https://en.wikipedia.org/wiki/Starlink

Section 2.2 http://www.ctio.noao.edu/noao/content/Basic-Optical-Parameters

Section 2.2 http://www.ctio.noao.edu/noao/content/Horizon-Limits

Section 2.2 $https : //www.google.com/search?q = Blanco + 4 - meter + telescoperlz = 1C5CHFA_enNZ901NZ901oq = Blanco+4-meter+telescopeaqs = chrome..69i57j33.451j0j7sourceid = chromeie = UTF - 8$

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Section 1.2 http://spaceref.com/astronomy/spacex-publishes-update-on-starlink-satellite-brightness-issue.html: :text=Based