Team 1274

How many satellites can be launched into orbit before astronomers on earth can no longer observe the night sky without interruption?

Summary

In this report, we calculate the number of satellites that can be launched into orbit before the view of the night sky is obstructed for astronomers. By simplifying the many complex variables involved in such a complex and open-ended question, we were able to define the parameters of which we were going to investigate. We used MATLAB to create a simulation that allowed us to test the different variables on the number of satellites which would interfere with the observation. Through this, we were able to interpolate the data to produce a model that would allow us to approximate the number of satellites which would interfere with the night sky. This value came out to be a total number of 106690 satellites or an additional 103,943 satellites on top of the current active 2,666 satellites.

Introduction

With the advent of affordable satellite launches, the world braces for a future enhanced by developments in satellite technology. The Starlink constellation - a project by SpaceX - aims to deliver high-speed internet access to even the remotest areas of the world; this will be achieved by placing the night sky with thousands of satellites. The company Amazon has similar plans, aiming to launch 3200 satellites for its own internet service. In the future, satellite constellations could add tens of thousands of satellites into Earth's orbit.

However, such plans endanger the field of astronomy. The proposed mega-constellations of satellites may prevent astronomers from taking pristine images of the night sky. Satellites appear as bright streaks across telescope images, resulting in a loss of valuable data about what lies behind those satellites.

If humanity wants to preserve the field of astronomy, it is important to consider at what point satellites will render scientists unable to survey the night sky without interruption. In this report, mathematical models will be used in order to estimate the number of satellites that will have to be launched for this to happen.

Defining the Problem

Due to the open nature of this question, we narrowed down the parameters of our investigation.

We consider "satellite" to be a manufactured object or vehicle intended to orbit the earth.[1] Therefore, in this report we will ignore the presence of space debris and any natural satellites (e.g Earth's moon). Since even one satellite's presence will affect the quality of the image produced, we shall consider that an "interruption" occurs when there is at least one satellite passing over a given region of space at any moment during an exposure time of 20 minutes. In order to reduce complexity, this region of space has been decided to equal the region of the sky (from Earth's surface) that completely covers the binary star system Alpha Centauri AB, the closest stars to Earth. We define this area as our "viewing circle".

We have chosen this relatively small "viewing circle" because it would be all too easy to claim that it is inevitable that at least one satellite will pass through a large area of the night sky and that this would constitute an interruption. Indeed, even today with the current number of satellites in Earth's orbit, images that capture large areas of the night sky are being disrupted by satellites.[2]

Furthermore, we only consider that astronomers cannot observe the night sky without interruption only if our definition of an "interruption" occurs in the majority of cases we try to view a certain area of the sky. In reality, if such interruptions were not common, scientists could simply choose a time that was suitable to take their observations of the night sky.

To investigate this question, we will develop a computer simulation in MATLAB to model the motion of satellites in Earth's orbit. This will allow us to gather a set of data for the number of satellites that interrupt our "viewing circle" for a given number of satellites. From this paired data, a model is developed for the relationship between mean number of interruptions and log(total number of satellites). Interpolating this model's line of best fit will result in our final answer - the number of satellites needed for the mean number of interruptions to be greater or equal to 1.

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[1]Retrieved August 1, 2020 from Merriam-Webster: https://www.merriam-webster.com/dictionary/satellite
[2] Retrieved August 1, 2020 from Vox
https://www.vox.com/science-and-health/2020/1/7/21003272/space-x-starlink-astronomy-light-pollution

Model of the Earth and its satellites

Calculation of Satellite Orbit

Due to the wording of the question, we assume the launches of satellites would be conducted in an educated manner such that the orbiting trajectories will not conflict and as such, collisions between satellites will not occur. Inactive satellites and space debris are not included in our investigation. Satellite orbits in Earth consists of three categories of high, medium and low-Earth orbits.[3] However, due to considerations of the possibility of 3-dimensional space for orbits in the practical sense of resource and time constraints of our team, we decided that orbits will all have very similar altitudes. Furthermore, in this investigation we will consider all orbits to be perfectly spherical, rather than an elliptical one.

Following these definitions and assumptions, we have calculated the average orbit altitude (above Earth) for the present 2,666 active satellites currently orbiting the Earth. Based on the Union of Concerned Scientists' database of the satellites currently orbiting Earth (updated April 1 2020)[4], we calculated the mean of the perigees and apogees of all individual satellites, and subsequently calculated the median of these values, obtaining an average of 615 km above Earth. Indeed, most orbiting objects and approved plans of satellite programs are concentrated in low Earth orbit, which makes our assumption of all satellite orbits to have an altitude of 615 km relatively justified.

Velocity of the Satellite

Using the following equation for the velocity of a satellite moving about a central body in circular motion:

$$v = \sqrt{\frac{G \times M_{central}}{R}}$$

Where G is $6.673 \times 10^{-11}\ Nm^2 kg^{-2}$, $M_{central}$ is the mass of the central body about which the satellite orbits, and R is the radius of orbit for the satellite.

R = height of the orbit + Earth's radius

$$v = \sqrt{\frac{6.673 \times 10^{-11} \times 5.972 \times 10^{24}}{(615+6371) \times 10^3}}$$

$v = 7552.768\ m\ s^{-1}$

$v = 7.55\ km\ s^{-1}$

Therefore, the velocity of all satellites in our investigation orbiting at 615 km above the surface of the Earth has a speed of $7.55\ km\ s^{-1}$.

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[3] Retrieved August 1, 2020 from https://earthobservatory.nasa.gov/features/OrbitsCatalog
[4] Retrieved August 1, 2020 from https://www.ucsusa.org/resources/satellite-database

Simulation of the Night Sky

Visible Angle of the Telescope

To define/calculate our viewing circle, we first have to calculate the viewing angle of which the telescope operates at. The diameter of Alpha Centauri AB is equal to 23 astronomical units (AU)[5], equal to $3.441 \times 10^9 km$, and the distance of this star from Earth is $4.12866 \times 10^{13} km$.

$$\text{Viewing angle} = tan^{-1}(\frac{3.441 \times 10^9}{4.12866 \times 10^{13}})$$
$$= 0.00477527°$$
$$= 0.0048° \ (2 \ s.f)$$

Note: for small values of tan(θ), θ is approximately equal to tan(θ), hence $tan(\frac{diameter/2}{distance}) \times 2$ $\approx tan(\frac{diameter}{distance})$

Exposure Time

The exposure time that was decided upon for our simulation was 20 minutes. This is equivalent to the exposure time of a telescope operating at a focal ratio of f/7.[6] This is a standard focal ratio for a mid-range telescope and acts as a lower bound for planetary imaging. A focal ratio below this, although it would require less time, would be insufficient to examine specific celestial bodies and hence does not meet the outlines we have defined for this problem. Our simulation ran for the same period of time as the exposure time for which we required our viewing circle to be clear of any interruptions.

The MATLAB functions below show the code used to obtain the mean number of satellites which enter our viewing circle. It does this by setting our viewing area in a large square which has an area of $6.13 \times 10^8 \ km^2$. This is equivalent to the surface area of a sphere which has a radius equal to the orbital radius of the satellites;

$$A = 4\pi r$$
$$A = 4\pi \times (615 + 6371)^2$$
$$A = 6.1329 \times 10^8 \ km^2$$
$$A = 6.13 \times 10^8 \ km^2$$

[5] Retrieved August 1, 2020 from https://nineplanets.org/alpha-centauri-system/
[6] Retrieved August 1, 2020 from https://starizona.com/tutorial/exposure-times/

Our viewing circle is at the centre of this large square, with satellites represented as straight lines. The getMean function simulates the night sky 100 times and outputs the mean number of satellites which cross our viewing circle, resulting in our table of values. In each simulation, for each satellite, a random initial position and direction is chosen and simulated over our exposure time. We calculate whether or not the satellite's path crosses the viewing area by testing if any point on this line is within the viewing area.

Variables

orbitRadius	The altitude at which the satellites orbit Earth in km
expoTime	The exposure time of our telescope i.e. the amount of time our viewing circle needs to be clear of satellites
speed	The speed at which the satellites orbit Earth in kms^{-1}
viewingAngle	The minimum angle required to view Alpha Centauri Binary Star
satelliteNumber	The number of satellites we simulate

MATLAB Code

```matlab
function numberOfIntersections = simulate(orbitRadius, expoTime, speed, viewingAngle, satelliteNumber)
    earthRadius = 6400;                      % radius of Earth
    totalRadius = earthRadius+orbitRadius;   % radius of Earth + altitude
    totalArea = 4*pi*totalRadius^2;          % surface area of sphere / square
    dimensions = sqrt(totalArea);            % dimensions of square
    midPoint = dimensions/2;                 % midpoint of square (also centre of viewing circle)
    viewingRadius = viewingAngle/360 * pi*totalRadius;  % radius of viewing circle

    numberOfIntersections = 0;      % initial number of intersections of satellites and our viewing circle is 0

    for i = 1:satelliteNumber       % simulate every satellite
        initialPosition = [rand*dimensions, rand*dimensions];    % choose a random initial position
        tmp = 2*(rand-0.5)*speed;                                % chooses a random direction vector in which the
        direction = [tmp, sqrt(speed^2 - tmp^2)];                % satellites travel, with magnitude equal to speed
        if rand <= 0.5
            direction(2) = -direction(2);
        end
        direction(1) = direction(1) - pi/43200 * totalRadius;   % accomodate for Earth's rotation

        finalPosition = initialPosition + expoTime*direction;   % calculate final position of satellite after expoTime

        % the following lines test if the satellite goes through the viewing circle
        % if it does, add one to the numberOfIntersections
        m = (finalPosition(2)-initialPosition(2))/(finalPosition(1)-initialPosition(1));
        c = finalPosition(2)-m*finalPosition(1);
        tmp1 = 2*midPoint - 2*m*(c-midPoint);
        tmp2 = sqrt((2*m*(c-midPoint)-2*midPoint)^2 - 4*(1+m^2)*(midPoint^2+(c-midPoint)^2-viewingRadius^2));
        tmp3 = 2*(1+m^2);
        x1 = (tmp1 + tmp2)/tmp3;
        x2 = (tmp1 - tmp2)/tmp3;
        if (isreal(x1) && x1 >= min(initialPosition(1), finalPosition(1)) && x1 <= max(initialPosition(1), finalPosition(1))) ..
                || (isreal(x2) && x2 >= min(initialPosition(1), finalPosition(1)) && x2 <= max(initialPosition(1), finalPosition
                || (initialPosition(1)-midPoint)^2+(initialPosition(2)-midPoint)^2 <= viewingRadius^2
            numberOfIntersections = numberOfIntersections + 1;
        end
    end
end
```

```
function getMean(satelliteNumber)
    orbitRadius = 615;
    expoTime = 1200;
    speed = 7.55;
    viewingAngle = 0.0048;
    data = zeros(1, 100);
    for i = 1:length(data)
        data(i) = simulate(orbitRadius, expoTime, speed, viewingAngle, satelliteNumber);
    end
    disp(mean(data));
end
```

Visual Representation of the Simulation

The coloured lines represent the paths of the simulated satellites. There is also a circle present representing the area of the viewing circle, but this is covered under the coloured lines in the diagrams below.


n = 100,000   n = 1,000   n = 1,000 (zoomed in)
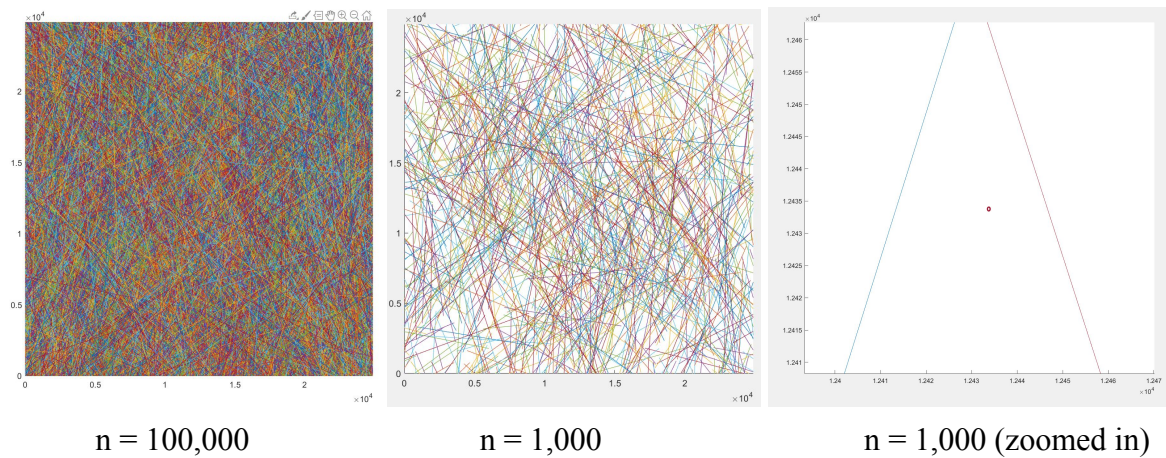
Discussion

Result from Simulation

The tabulated data obtained from our simulation and the scatter plot graph plotted accordingly are shown below.

Number of Satellites	Interruptions for 100 simulations	Log(number of Satellites)
1	0	0.000
5	0	0.699
10	0	1.000
50	0	1.699
100	0	2.000
500	0.01	2.699
1000	0.02	3.000
5000	0.04	3.699
10000	0.08	4.000
50000	0.42	4.699
100000	1.03	5.000
500000	4.38	5.699
1000000	8.77	6.000
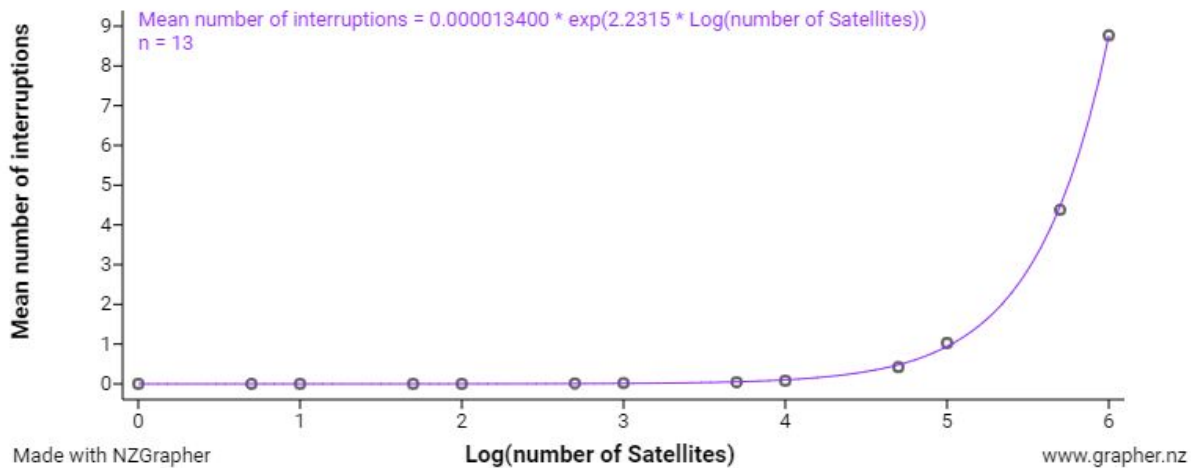
A model for the mean number of interruptions for any given number of satellites can be obtained from the line of best fit:

$$0.0000134 \times e^{2.2315 \times log(number\ of\ satellites)}$$

We use this line to calculate the number of satellites in which we have a mean number of one interruption.

$$0.0000134 \times e^{2.2315 \times log(number\ of\ satellites)} = 1$$

Solving for number of satellites:

Number of satellites = 106,689.7557

= 106690 (rounded up to the nearest whole number)

Assumptions

1. Satellites to be considered a point mass; the impact of this assumption is likely to be limited due to the fact that the size of most satellites are relatively small.
2. The probability of collisions for the satellites is considered to be 0..
3. All satellites follow the orbit of the same altitude of 615 km above the surface of Earth.
4. The velocity of the satellites is constant as a result of the equal altitudes.
5. All satellites are released instantaneously into space.
6. We assume that satellites only contribute optical inference, and do not contribute radio wave interference.
7. We assume that all telescopes have constant exposure time.

Limitations

1. It is inaccurate to represent the motion of satellites in 3D curved space in 2D space; in our model we simulate a 3-D approximation in a 2-D plane. Although in the simulation our 'rectangle' has the same surface area of the Earth's, it still lacks the consideration of curvature in the movement.
2. Even if no satellites are within our viewing circle, there will always be some form of interference which will affect the observation. In our model, we assume that satellites do not produce any additional interference apart from blocking the target.
3. In our model, the presence of even one satellite is unacceptable and considered to be an interference. In reality, minor interference can be edited out and dealt with.

Conclusion

We have calculated from our model that we estimate that there needs to be 106690 satellites in the Earth's low orbit for astronomers to no longer observe the night sky without interruption. Considering that there are 2,666 artificial satellites currently in orbit[7], we perform the following calculation:

Number of more satellites to be launched = 106609 - 2666 = 103943

Therefore, we are happy to present that our predicted estimate for the number of satellites that need to be launched is 103,943 for astronomers to no longer observe the night sky without interruption.

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[7] Retrieved August 1, 2020 from https://www.ucsusa.org/resources/satellite-database