

New Zealand's Next Top Engineering Scientist

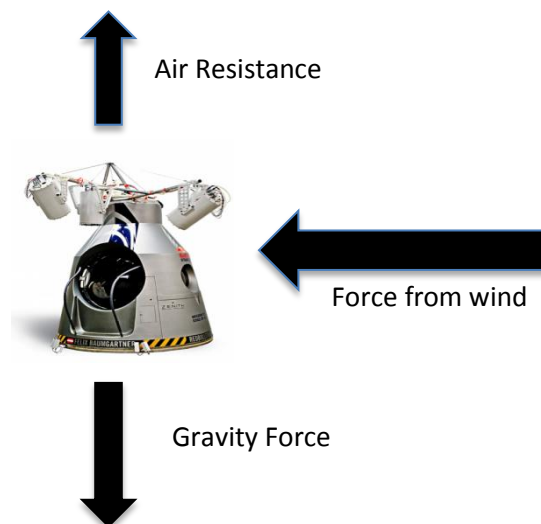
Team 2005

Introduction

In order to determine the area at which the capsule lands, we had to determine the size of the capsule, its dimension and mass and how the speed of the wind will affect its trajectory when it descends. We have presumed that the capsule was released over a large land mass that isn't mountainous, meaning that mountains or any other obstacles like trees could not impede or channel the wind, therefore affecting the speed. In our investigation we have accounted for the fact that wind speeds will alter, as well as the gravitational strength, depending on the atmospheric layering. The area of the direction that they would deviate from the original release point will therefore depend on the time it spends on each atmospheric layer. Because the different atmosphere's will have different density of air particles, the proportionality of the forces acting on the capsule means that terminal velocity will constantly change throughout its journey as it enters different atmospheric conditions. We have also considered the effects of weather on the capsule and since the weather contributes to the speed of the wind, we have made one final assumption that the weather on the launch day will be clear. This assumption can be justified because the weather balloon cannot be launched in heavy winds.¹ All these factors contribute to the final displacement of the capsule and through this we can find the velocity of all the wind speeds that would affect and shift the capsule's horizontal trajectory. For this investigation we have used the free-falling model.

How will the forces affect the trajectory of the capsule?

Through our research we have determined the size of the capsule as diameter of 8 feet and length of 11 feet with the mass as 1315 kg². Based on our knowledge we know that the forces acting on the capsule is gravitational force (which acts downwards), air resistance (which opposes gravitational force) and wind (which will act on the capsule's horizontal vector component). All these components, horizontal and vertical, will contribute to the final displacement of the capsule and therefore enable us to determine the maximum area it could land at.³



¹ <http://abcnews.go.com/Technology/skydiver-felix-baumgartner-plans-daredevil-jump-balloon-edge/story?id=16838365>

² <http://www.redbullstratos.com/technology/capsule/>

³ www.flickr.com/photos/airports_fai/764222414

Modelling our plan

In order to determine the area, we first found the different gravitational force acting on the capsule base on their layer in the atmosphere. To find the terminal velocity which the capsule experiences, we found the air resistance that would oppose the gravitational force by concluding the density of air particles in each layer which will act against the body to produce air resistance. Once terminal velocity is found, we assumed that this is the velocity that the capsule is travelling until it reaches a new terminal velocity. We are then able to find the time which the capsule travels within the area of constant speed and the specific force from the wind that acts on it affecting its motion. From this force we can work out the horizontal acceleration of the capsule and hence horizontal motion of the capsule. With this we can determine the distance which the capsule could land.

Acceleration due to gravity in the different layers of atmosphere

We have taken into account the different gravitational strength in the different atmosphere that would result in different values of acceleration.

At 37 km

$$g_{lower} = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.37 \times 10^6 + 37 \times 10^3)^2} = 9.7 \text{ ms}^{-2}$$

At 20 km

$$g_{lower} = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.37 \times 10^6 + 20 \times 10^3)^2} = 9.75 \text{ ms}^{-2}$$

At 12 km

$$g_{lower} = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.37 \times 10^6 + 12 \times 10^3)^2} = 9.78 \text{ ms}^{-2}$$

At 7 km

$$g_{top} = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.37 \times 10^6 + 7 \times 10^3)^2} = 9.79 \text{ ms}^{-2}$$

At 0 km

$$g_{top} = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.37 \times 10^6)^2} = 9.81 \text{ ms}^{-2}$$

Average for Upper Stratosphere:

$$\frac{9.75 + 9.7}{2} = 9.725 \text{ ms}^{-2}$$

Average for Lower Stratosphere:

$$\frac{9.75 + 9.78}{2} = 9.765 \text{ ms}^{-2}$$

Average for Upper Troposphere:

$$\frac{9.79 + 9.78}{2} = 9.785 \text{ ms}^{-2}$$

Average for Lower Troposphere:

$$\frac{9.79 + 9.81}{2} = 9.8 \text{ ms}^{-2}$$

How will the different atmospheric layering affect the wind speed?

The density at each layer of the atmosphere will affect the proportionality of the forces acting on the capsule. As velocity increases, there will be more air particles hitting the body, therefore in order to calculate terminal velocity we would require the density of air particles to find the air friction opposing acceleration due to gravity. Air density decreases as altitude increases. Aerodynamic forces depend on air density which differs in each atmosphere and using this theory we can determine the variation of properties in each atmosphere. ⁴

Earth Atmosphere Model for Troposphere

In the troposphere, the temperature decreases linearly and pressure increases exponentially. It runs from the surface of the Earth to 36,152 feet and the model used is:

$$\begin{aligned} T &= 59 - 0.00356 \times h \\ T &= 59 - (0.00356 \times 36152) \\ T &= -69.70112 \text{ }^\circ\text{F} \end{aligned}$$

...where T = temperature in (°F)

h = altitude (ft)

Using the temperature found for the given altitude we can determine the pressure...

$$\begin{aligned} p &= 2116 \times [(T + 459.7) \div 518.6]^{5.256} \\ p &= 2116 \times [-69.70112 + 459.7] \div 518.6]^{5.256} \\ p &= 473.135 \text{ lbs/sq ft} \end{aligned}$$

...where p = pressure (lbs/sq ft)

⁴ Earth Atmosphere Model –English Units. Retrieved September 22,2012 from <http://exploration.grc.nasa.gov/education/rocket/atmos.html>

With the pressure given we can find the density of the specific atmosphere which in this case is the troposphere

$$\rho = \frac{p}{[1718 \times (T + 459.7)]}$$

$$\rho = \frac{473.135}{[1718 \times (-69.70112 + 459.7)]}$$

$$\rho = 7.0615 \times 10^{-4} \text{ slugs/cu ft}$$

...where p = density (slugs/ cu ft)

Given that 1 slugs/cu ft = **515.378818 kg / m³**

$$\rho = \mathbf{0.3639 \text{ kg/m}^3}$$

...where ρ = density (kg/m³)

Earth Atmosphere Model for Lower Stratosphere

The lower stratosphere runs from 36,152 feet to 82,545 feet and to find altitude for the equation, we calculated the average between 36,152 and 82,545. Its temperature is constant and the pressure decreases exponentially and the model is:

$$T = -70$$

Given the value of temperature we can find out the pressure...

$$p = 473.1 \times 10^{(1.73 - 0.000048 \times h)}$$

The average value between the lower stratosphere is...

$$h = \frac{36,152 + 82,545}{2}$$

$$h = 59,348.5 \text{ ft}$$

So we can substitute this average altitude

$$p = 473.1 \times 10^{(1.73 - 0.000048 \times h)}$$

$$p = 473.1 \times 10^{(1.73 - 0.000048 \times 59,348.5)}$$

$$p = 35.9936 \text{ lbs/sq ft}$$

Now we can find the density...

$$\rho = \frac{p}{[1718 \times (T + 459.7)]}$$

$$\rho = \frac{35.9936}{[1718 \times (-70 + 459.7)]}$$

$$\rho = 5.376 \times 10^{-5} \text{ slugs/cu ft}$$

In kg / m³ ...

$$\rho = 0.0277 \text{ kg/m}^3$$

Earth Atmosphere Model for Upper Stratosphere

The upper stratosphere model is used for altitudes above 82,345 feet where the temperature increases slightly and the pressure decreases exponentially. To find the altitude we have taken the average value between the capsule's release point and the minimum value of the upper stratosphere.

$$T = -205.05 + 0.00164 \times h$$

Where h is....

$$h = \frac{120000 + 82345}{2}$$

$$h = 101172.5 \text{ ft}$$

So to find the temperature....

$$T = -205.05 + 0.00164 \times h$$

$$T = -205.05 + 0.00164 \times 101172.5$$

$$T = -39.121 \text{ }^\circ\text{F}$$

We can now find the pressure of the upper stratosphere....

$$p = 51.97 \times \left[\frac{(T + 459.7)}{389.98} \right]^{-11.388}$$

$$p = 51.97 \times \left[\frac{(-39.121 + 459.7)}{389.98} \right]^{-11.388}$$

$$p = 21.987 \text{ lbs/sq ft}$$

With this pressure we can find the density of air in the upper stratosphere density...

$$\rho = \frac{p}{[1718 \times (T + 459.7)]}$$

$$\rho = \frac{21.987}{[1718 \times (-39.121 + 459.7)]}$$

$$\rho = 3.043 \times 10^{-5} \text{ slugs/cu ft}$$

In kg/m³

$$\rho = 0.01567 \text{ kg/m}^3$$

By calculating the density of each atmospheric layer, we can calculate the terminal velocity at each layer of the atmosphere using the equation⁵:

$$D = C_d \rho \frac{V^2 A}{2}$$

...where C_d is a constant –in our case it is 1

A is the area of the surface that is perpendicular to the motion

⁵ <http://www.grc.nasa.gov/WWW/k-12/airplane/falling.html>

V is velocity

ρ is the air pressure of the corresponding layers of atmosphere

This assumes that the force of gravity is equal to the force of air resistance so acceleration is zero.

What is the terminal velocity each of the atmospheric layer?

Since we have already calculated the gravitational force which acts to accelerate the capsule downwards and we have just calculated the density of air in each layer of atmosphere, we are able to find the air resistance that opposes the gravitational force so that both values are equal and terminal velocity is achieved.

These are the terminal velocity for each level. We calculated three terminal velocities: troposphere, lower stratosphere and the upper stratosphere. In these calculations, we used the co-efficient of drag for a long circular cylinder end-on shape because as the capsule is falling down, the surface area interacting with air friction is similar to the surface area of a long circular cylinder end-on shape. Also, because there is no known co-efficient for a bell shaped object we have omitted out the top part of the capsule.

The surface area of the capsule is:

$$A = \pi r^2 = \pi(1.2)^2 = 4.5m^2$$

Troposphere

$$F_{drag} = F_{weight}$$

$$F_{drag} = \frac{C_d \rho A v^2}{2}$$

$$F_{weight} = ma$$

$$\frac{C_d \rho A v^2}{2} = ma$$

$$\frac{1 \times 0.3639kgm^2 \times 4.5m^2 \times v^2}{2} = 1315kg \times 9.795m/s$$

$$v^2 = 15731.34$$

$$v = 125.42m/s$$

Upper Stratosphere

$$F_{drag} = F_{weight}$$

$$\frac{C_d \rho A v^2}{2} = ma$$

$$\frac{1 \times 0.01569 \text{kgm}^2 \times 4.5 \text{m}^2 \times v^2}{2} = 1315 \text{kg} \times 9.725 \text{m/s}$$

$$v^2 = 362251.257$$

$$v = \mathbf{601.87 \text{m/s}}$$

Lower Stratosphere

$$F_{drag} = F_{weight}$$

$$\frac{C_d \rho A v^2}{2} = ma$$

$$\frac{1 \times 0.0277 \text{kgm}^2 \times 4.5 \text{m}^2 \times v^2}{2} = 1315 \text{kg} \times 9.765 \text{m/s}$$

$$v^2 = 206032.491$$

$$v = \mathbf{453.9 \text{m/s}}$$

The following are the calculations we used to find the distance each capsule travelled horizontally in each atmospheric layer. To do this, we assumed that the acceleration of the capsule in each layer is constant. In all the following equations and calculations, horizontal velocity was used, unless stated otherwise.

Upper Stratosphere

We first found the time the capsule spent in the upper stratosphere. The velocity that was used in this equation was the vertical velocity.

$$d = V_i t + \frac{1}{2} a t^2$$

$$17,000 = 0 + \frac{1}{2} (9.725) t^2$$

$$t^2 = \sqrt{\frac{17000}{0.5 \times 9.725}}$$

$$t = 59 \text{s}$$

Then, we found the wind the force would exert on the capsule, where A is the area, P is the wind pressure and C_d is the co-efficient of drag.

Area the wind is acting on:

$$A = \text{base} \times \text{height} = 3.35 \times 2.44 = 8.17m^2$$

In this area, we use the assumption that the capsule is shaped like a circular cylinder. And a circular cylinder will have a co-efficient of 1.2.⁶ We used different C_d values for horizontal and vertical velocity because we felt that a circular cylinder would better model the capsule's horizontal velocity.

We also had to calculate the pressure at this layer of atmosphere, which we found to be:

$$P = 0.00256 \times v^2, \text{ in miles per hour}^7$$

$$F_{wind} = A \times P \times C_d = 8.17 \times (0.00256 \times 179^2) \times 1.2 = 804N$$

$$a = \frac{F}{m} = \frac{804}{1315} = 0.61ms^{-2} \text{ horizontally}$$

$$d = v_i t + 0.5(at^2) = 0 + 0.5(at^2) = 0.5 \times 0.61 \times 0.59^2 = 1064m \text{ horizontally}$$

Next, we calculate the final velocity of the upper stratosphere, which will be the initial velocity of the lower stratosphere.

$$v_f = v_i + at = 0 + 0.61 \times 59 = 36m/s$$

Lower Stratosphere

When the capsule enters the lower stratosphere, gravity force increase, but the lift force increases proportionally more so that the capsule decelerates. We know this because the terminal velocity in the lower stratosphere is lower than the higher stratosphere, so it must have decelerated. When this

⁶ <http://www.crcnetbase.com/doi/abs/10.1201/9781420065572-a3>

⁷ <http://k7nv.com/notebook/topics/windload.html>

happens, velocity decreases, so air resistance, which is proportional to v^2 , will decrease also, until $F_g = F_{\text{drag}}$ and a new terminal velocity is established. We will assume that this happens in a negligible amount of time so in the lower stratosphere, velocity is constant at 453.9m/s.

$$d = vt$$

$$t = \frac{d}{v} = \frac{8000}{453.9} = 17.6s$$

$$F_{\text{wind}} = A \times P \times C_d = 8.17 \times (0.00256 \times 100^2) \times 1.2 = 251N$$

$$a = \frac{F}{m} = \frac{251}{1315} = 0.191 \text{ ms}^{-2} \text{ horizontally}$$

So the distance travelled horizontally during this time in this layer of the atmosphere is:

$$d = v_i t + 0.5(at^2) = 36 \times 17.6 + 0.5 \times 0.191 \times 17.6^2 = 663.2m \text{ horizontally}$$

Next, we calculate the final velocity of the lower stratosphere, which will be the initial velocity of the upper troposphere.

$$v_f = v_i + at = 36 + 0.191 \times 17.6 = 39.36m/s$$

Upper Troposphere

When the capsule enters the upper troposphere, lift force increases, but the gravity force increase proportionally more so that the capsule accelerates. We know this because the terminal velocity in the upper troposphere is higher than the lower stratosphere, so it must have accelerated. When this happens, velocity increases, so air resistance, which is proportional to v^2 , will increase also, until $F_g = F_{\text{drag}}$ and a new terminal velocity is established. We will assume that this happens in a negligible amount of time so in the lower stratosphere, velocity is constant at 125.42m/s.

$$t = \frac{d}{v} = \frac{5000}{125.42} = 39.86s$$

$$F_{\text{wind}} = A \times P \times C_d = 8.17 \times (0.00256 \times 300^2) \times 1.2 = 2259N$$

$$a = \frac{F}{m} = \frac{2259}{1315} = 1.71 \text{ ms}^{-2} \text{ horizontally}$$

So the distance travelled horizontally during this time in this layer of the atmosphere is:

$$d = v_i t + 0.5(at^2) = 39.36 \times 40 + 0.5 \times 1.72 \times 40^2 = 2950.4m \text{ horizontally}$$

Next, we calculate the final velocity of the upper troposphere, which will be the initial velocity of the lower troposphere.

$$v_f = v_i + at = 39.36 + 1.72 \times 40 = 108.16m/s$$

Lower Troposphere

$$t = \frac{d}{v} = \frac{7000}{125.42} = 55.81s$$

$$F_{wind} = A \times P \times C_d = 8.17 \times (0.00256 \times 231^2) \times 1.2 = 1339.9N$$

$$a = \frac{F}{m} = \frac{1339.9}{1315} = 1.02 \text{ ms}^{-2} \text{ horizontally}$$

So the distance travelled horizontally during this time in this layer of the atmosphere is:

$$d = v_i t + 0.5(at^2) = 108.6 \times 55.81 + 0.5 \times 1.02 \times 55.81^2 = 7623.3m \text{ horizontally}$$

So, the final radius of the search area will be all the distances added together:

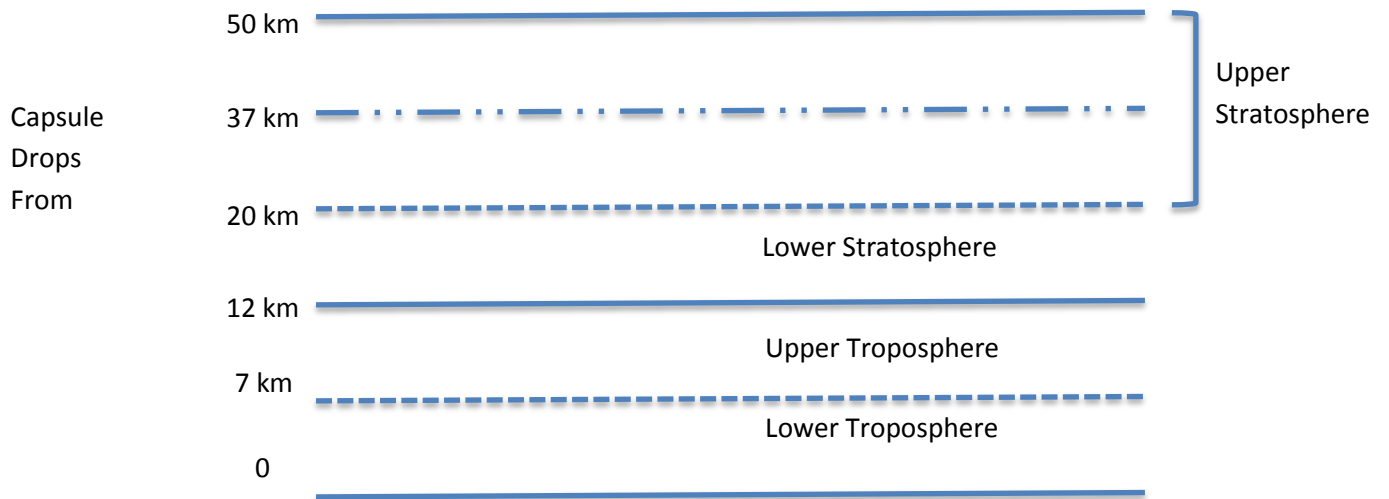
$$r = 7323.3 + 1064 + 663.2 + 2950.4 = 12300.9 = 12300m$$

So, the final area that the capsule could be found in is:

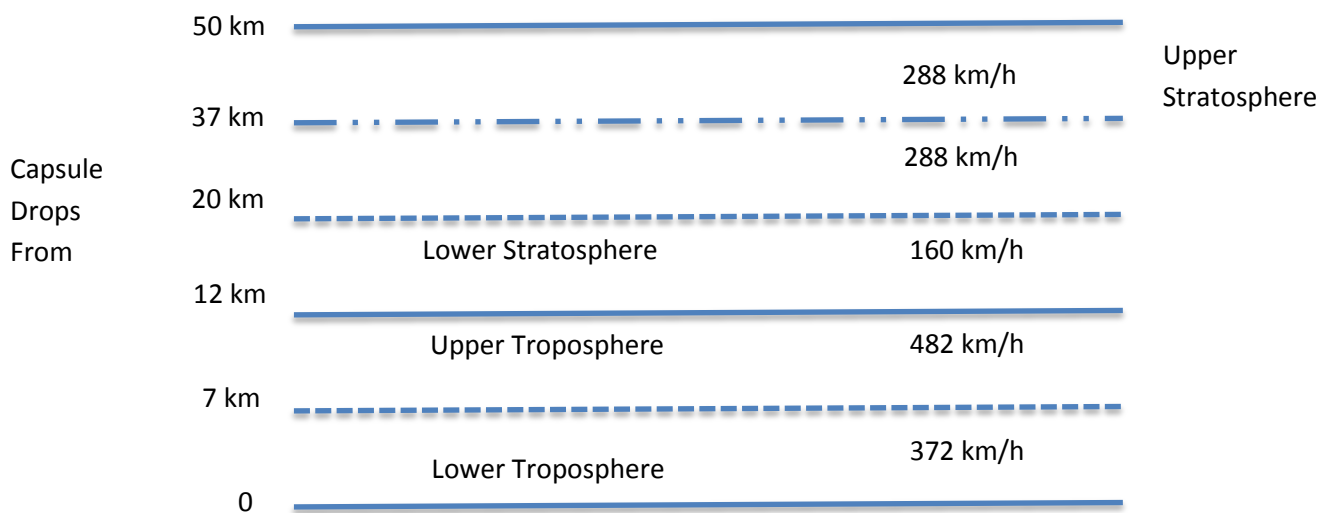
$$A = \pi r^2 = \pi \times (12.3)^2 = 475.3km^2$$

Conclusion and Validity of Assumptions

In conclusion, the area the people should search is 475.3 km². We have used a circle to model the area which the capsule could land because the effects of wind could be in any direction and there was no way of finding out which direction of the wind would act at a certain time. We have assumed the boundaries of different altitudes of the different layers of the atmosphere because it is not a quantity that is easily measured.



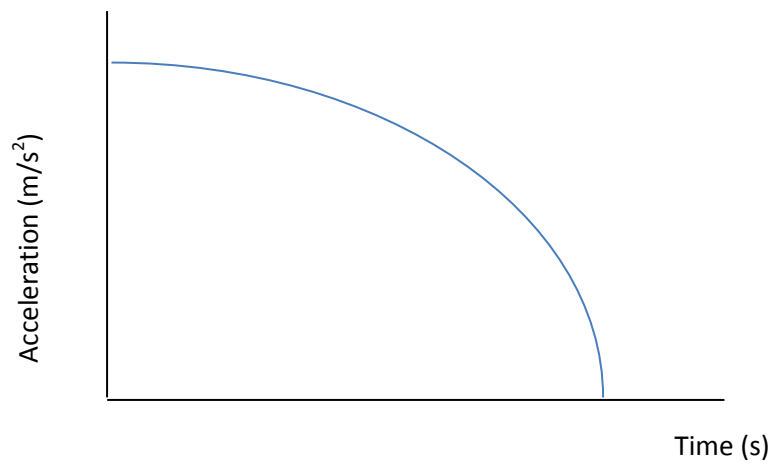
We have also estimated the different wind speeds in the different altitudes and layers of atmosphere.⁸ We used maximum wind speeds in each layer in every atmosphere because it is better to overestimate our values than to underestimate it.⁹



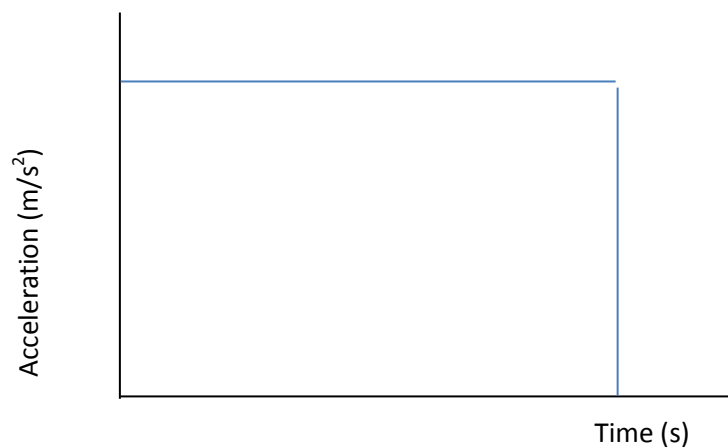
⁸ <http://www.mountwashington.org/about/visitor/recordwind.php>

⁹ http://centennialofflight.gov/essay/Dictionary/Jet_Stream/DI68.htm

In real life:



However, for this part of the investigation we have assumed that acceleration was constant until it reached terminal velocity when it became zero.



We made this assumption because the change in acceleration happened so quickly that the difference between values calculated from each of the two situations is negligible.

In our calculations we did not take into account effects of a parachute, however in real life, a parachute would have been used greatly increased the surface area upon which drag force would act and thus affecting the time in each layer and hence the horizontal distance and the radius of the search area.

Summary:

In this investigation, we were asked to find the size of the search area of a capsule that was dropped from the edge of space would have. This capsule did not have a tracking device to use and we therefore, had to use mathematical and science models to estimate and approximate the size of the capsule's search area. We used free-falling as a model and from this we were able to calculate that the approximate area is 475.3kmsquared.

To find this, we had to research and calculate several factors that would eventually all add up to give us this area. The first thing we did was to research a little about the problem and the situation. From our research we were able to deduce the size of the capsule and the different altitude of the layers of the atmosphere. With these altitudes, we were able to find the wind speeds, air pressure and the density of air pressure of the different layers. We then use these to find the terminal velocity of the capsule and all these factors were used to find the time the capsule spent at each layer of atmosphere. And using this time, we worked out the distance the wind exerted on the capsule to move it horizontally. And eventually, with all these factors, we were able to find the radius (the distance the capsule travelled from it's original position) and we used this to find the size of the search area, which we found to be 475.3kmsquared.

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