

1097

“If Mount Taranaki erupted, how much would it cost the aviation industry?”

Introduction

The true cost to the aviation industry is hard to estimate, there are many factors beyond simple costs and lost ticket sales. For example, long term effects on public perception of New Zealand routes, costs of stranded passengers, et cetera. For the purpose of this report costs will be defined as lost revenue from routes that cannot be flown. The primary revenue loss event that occurs during the eruption of the volcano will be the spreading of a large ash cloud which disrupts air traffic at various altitudes.

Effect of Volcanic Ash on Jet Engines

A flying plane has four main forces acting on it - gravity, lift, drag and thrust. Jet engines provide the thrust aspect of this model. A fan at the front of the engine sucks in air in large volumes. This air is then compressed by the rapidly spinning blades of the compressor. Fuel is sprayed into the compressed air and is combusted. The quickly expanding gas blows out through the rear nozzle.¹ According to Newton’s third law of motion, “if one object A exerts a force F_A on a second object B, then B simultaneously exerts a force F_B on A, and the two forces are equal and opposite: $F_A = -F_B$.”² Thus a forward force is generated.

The three main components of volcanic ash are fragments of rock, crystalline material and volcanic glass. Volcanic glass has a melting point of lower than the temperature in the combustor in the gas turbine which means that any volcanic glass that gets into the gas turbine is likely to melt. However, the combustor and turbine components are cooled since they are made out of components that have lower melting points than the gas temperature in the engine core. Molten gas that touches these surfaces is likely to freeze and thus clog the air nozzle. The compressor will no longer be able to hold all the compressed gas and it will burst out the front of the engine, accompanied by a ball of flame. This is known as an engine surge. The ball of flame will starve the flame in the engine combustor known as a “flame-out”. This means that the engine shuts down and stops providing thrust.³

Although it is possible for planes to fly under or over the ash cloud, it is unlikely to be economically viable, or indeed socially viable (considering recent aviation disasters) from the perspective of the airline to do so. Furthermore, in such conditions, the CAA (Civil Aviation Authority) has guidelines in place⁴ which would not allow the planes to fly in volcanically active areas.⁵

¹ Dr. Shaw. R. J (2014) “How Does a Jet Engine Work” Retrieved from <http://www.grc.nasa.gov/WWW/k-12/UEET/StudentSite/engines.html>

² Sir Newton. I (1687) “Philosophiæ Naturalis Principia Mathematica” Retrieved from http://books.google.co.nz/books?id=Tm0FAAAAQAAJ&pg=PA19&redir_esc=y#v=onepage&q&f=false

³ Author Unknown (2014) “Volcanic ash and aviation safety” Retrieved from http://en.wikipedia.org/wiki/Volcanic_ash_and_aviation_safety#Immediate_hazards_to_aircraft

⁴ Lechner, Peter (2014) “Living with Volcanic Ash Episodes in Civil Aviation” Retrieved from http://www.caa.govt.nz/meteorology/Volcanic_Ash_Advisory_System.htm

⁵ Olsen. J (2011) “Explainer: Why can’t planes fly through volcanic ash?” Retrieved from <http://theconversation.com/explainer-why-cant-planes-fly-through-volcanic-ash-1959>

Extent of Ash Cloud Produced

We made an assumption that the eruption produced by Mt Taranaki would be on the order of a category 5 on the Volcanic Explosivity Index, considering that there hasn't been a major eruption of the volcano since 1655, and the frequency of major eruptions of a similar scale is given as being once every five hundred years. The eruption in 1655 also is an example of a transition from an effusive eruption style to a more explosive style.⁶ We are assuming the eruption plume's maximum height would be around 90,000 feet, or roughly 30km into the air, based on the VIA (Volcanic Explosivity Index) figure given.⁷

Mt Taranaki last erupted in 1655 and the expected time between major eruptions has been given as 500 years⁸, further improving the chances that the eruption would be major. Assuming a worst-case scenario, the type of eruption likely to happen if Mt Taranaki were to erupt is expected to be on the order of a category 5 on the Volcanic Explosivity Index, which puts the eruption plume's maximum height near to 30km.

Mathematical model of ash cloud movement

We have derived a mathematical model estimating the distance that the eruption's resulting ash cloud would travel. This model is based on the terminal velocity of the fall of the particles as being a function of Reynold's equation, the drag coefficient and gravitational settling.⁹

$$R_e = \frac{dv_t \rho}{\eta}$$

$$C = \frac{24}{R_e}$$

$$V\sigma g = \frac{1}{2} C \rho A v_t^2$$

Where d = particle diameter, v_t = terminal velocity, ρ = atmospheric density, η = viscosity of the atmosphere, R_e = Reynold's number, C = drag coefficient, A = particle cross sectional area, g = gravitational acceleration, σ = particle density and V = particle volume.

The model uses spherical particles of density 2.510^6 gm^{-3} with diameters ranging from 0.3 to 30 microns. This particle distribution was derived from aircraft sampling of Mount St. Helens and Redoubt Volcano ash clouds¹⁰, eruptions similar in scope and scale to the Mt Taranaki eruption in question.

Terminal velocity is calculated as⁹

$$v_t = \frac{(d^2 \sigma G)}{\eta}$$

Where d is diameter, σ is particle density, and G is the gravity constant, 980 cms^{-2}

⁶ <http://www.volcanodiscovery.com/taranaki.html>

⁷ https://en.wikipedia.org/wiki/Volcanic_Explosivity_Index

⁸ https://en.wikipedia.org/wiki/Mount_Taranaki

⁹ http://www.geo.mtu.edu/~gbluth/Teaching/GE4150/lecture_pdfs/L9a_ashfall.pdf

¹⁰ Hobbs, P. V., L. F. Radke, J. H. Lyons, R. J. Ferek, D. J. Coffman, and T. J. Casadevall, 1991: Airborne measurements of particle and gas emissions from the 1990 volcanic eruptions of Mount Redoubt, J. Geophys. Res., 96, 18 735-18 752.

Viscosity Function

The viscosity of the atmosphere depends on the temperature of the air. It is given by the Sutherland Equation, where T is temperature in Kelvin, $b = 1.458 \times 10^{-6}$ and $S = 110.4\text{K}$.¹¹

$$\mu = \frac{bT^{3/2}}{T + S}$$

The temperature varies at different altitude, and can be modelled as a function of h =altitude in metres, measured in Celsius.¹²

For $h < 11000\text{m}$ the temperature is $15.04 - 0.00649h$, giving a viscosity function of

$$\frac{1.458 \times 10^{-6}(288.19 - 0.00649h)^{1.5}}{398.59 - 0.00649h}$$

with temperatures given in K.

When $11000\text{m} < h < 25000\text{m}$ the temperature is -56.46 Celsius, giving a viscosity of 0.0000142183 .

At $25000\text{m} < h < 50000\text{m}$ the temperature is given as $-131.21 + 0.0299h$ Celsius,

$$\frac{1.458 \times 10^{-6}(141.94 + 0.0299h)^{1.5}}{0.0299h + 252.34}$$

Below is the table of calculations in an Excel spreadsheet calculating terminal velocity for each of the three height intervals, and from that, fall time in hours is calculated.

First, here's the formula when applied to the minimum particle size of 0.3 microns:

	Diameter	Diameter	part density	gravity	atm visc	const	term vel	vert dist	fall time
level	cm	cm^2	g/cm^3	cm/s^2	g/cm s	const	cm/s	cm	hrs
m	d	d^2	sigma	g	n	const	T		
25000 to 30000	3.00E-05	9.00E-10	2.51	981	2.67E-04	18	8.29E-03	500000	16753.96126
11000 to 25000	3.00E-05	9.00E-10	2.51	981	1.42E-04	18	1.56E-02	1400000	24951.00079
0 to 11000	3.00E-05	9.00E-10	2.51	981	1.05E-04	18	2.11E-02	1100000	14471.86926
								hrs	56176.83131
								days	2340.701305
								yrs	6.412880287

This means that for all of the particles to fall of all the sizes given in the past data measurement, it will take ~6.4 years. However, we can quite safely make the assumption that these particles are too small to make a substantial impact on the operation of aircraft jet engines.

Here's the formula when applied to the minimum bound of 75% of the range given above.

¹¹ http://www-mdp.eng.cam.ac.uk/web/library/enginfo/aerothermal_dvd_only/aero/fprops/propsoffluids/node5.html

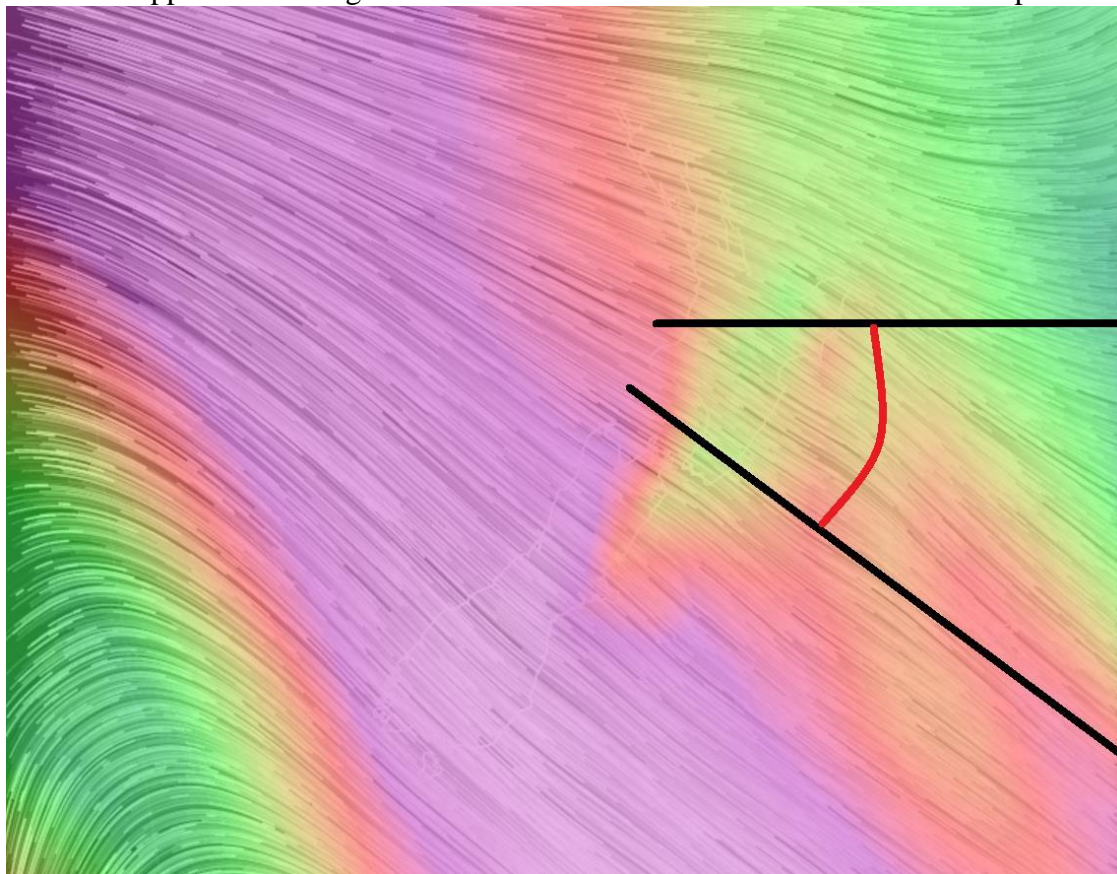
¹² <http://exploration.grc.nasa.gov/education/rocket/atmosmet.html>

	Diameter	Diameter	part density	gravity	atm visc	const	term vel	vert dist	fall time	windspee	hor dist
level	cm	cm^2	g/cm^3	cm/s^2	g/ cm s	const	cm/s	cm	hrs	km/h	km
m	d	d^2	sigma	g	n	const	T				
25000 to 30000	7.73E-04	5.97E-07	2.51	981	2.67E-04	18	5.50E+00	500000	25.26754455	160	4042.807
11000 to 25000	7.73E-04	5.97E-07	2.51	981	1.42E-04	18	1.03E+01	1400000	37.62993804	108	4064.033
0 to 11000	7.73E-04	5.97E-07	2.51	981	1.05E-04	18	1.40E+01	1100000	21.82579962	72	1571.458
									hrs		9678.298
									days		3.530136759
									yrs		0.009671608

At the bottom, it can be seen that it would take 3.53 days for all of the particles within that size range to fall.

We have used approximate wind speed data obtained to calculate, based on the time taken to fall, how far the particles will travel horizontally from the source.

Below is a diagram below showing the wind patterns of New Zealand.¹³ Over it, we have drawn the approximate range of this worst-case ash cloud based on these wind patterns.

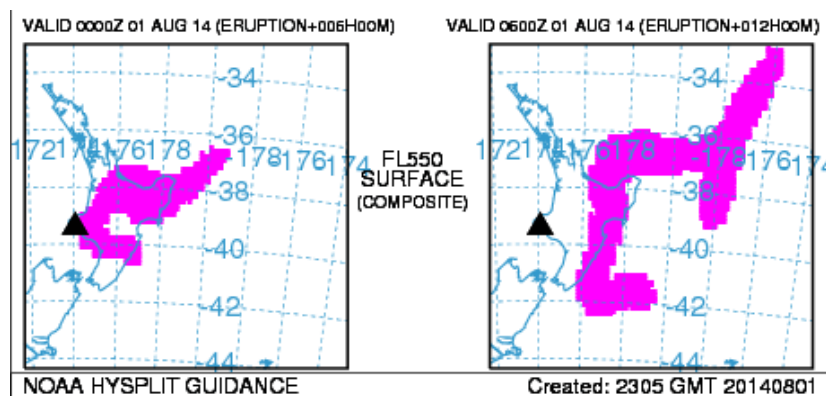


The red line represents the distance travelled by the largest particles (30 microns). The black line approximately represents the dispersion of the ash cloud. Due to the prevailing wind current through New Zealand it is not that wide. The smallest important particles will travel 9,678.3 km however we deduced that due to the wind currents around Antarctica the ash particles will not travel much further than that and will continue to circle Antarctica until they fall. The effect on the aviation industry by ash clouds in such a location will be negligible.

¹³ <http://wairoa.net/weather/windmap.htm>

NOAA alternate simulation

We conducted a second simulation, using the NOAA HYSPLIT simulation. We used archive weather data and the same parameters as given in the mathematical model. Diagrams are given below for the simulation, based on these parameters, which we ran on the NOAA Air Resources Laboratory website¹⁴:



The diagram above illustrates the extent of a worst-case ash cloud 5 hours after eruption and 12 hours after eruption.

‘Worst-case’ is defined here as a situation where the ash cloud is blown northward, caught in typical east-coast winds, spreading out over much of the north island.

From the two models we used, we can say with a high probability that both of the North Island’s major airports, Auckland International Airport and Wellington Airport, would close down completely.

Economic Impact

Calculating the total flight numbers into Auckland & Wellington, and the money spent on those flights, per day, is a difficult exercise. Unable to find exact figures we decided to analyse easier-to-find data to create a ballpark estimate for international air travel revenue.

32 international planes arriving into Auckland¹⁵ were tallied, making note of the origin port. Using this data we estimated the share major overseas ports played in sending passengers to New Zealand. We made the assumption that they would receive roughly equal amounts of passengers (ie that the exchange was two way).

Combined with passenger statistics released by the Auckland International Airport, this allowed us to calculate the number of passengers coming from each port. Online ticketing service, Hipmunk , was used as it provides actual ticket costs (as opposed to heavily

¹⁴ http://www.arl.noaa.gov/HYSPLIT_ashinterp.php

¹⁵ <http://www.aucklandairport.co.nz/>

discounted/opportunistic tickets typically advertised), to estimate the price of a ticket from each port. Combined this data gives us an estimate of the total international revenue per day.

Revenue of Primary Components

International Airline Revenue (Auckland & Wellington)

For competitive reasons revenue of specific routes is often not made publicly available by airlines. As such, making an estimate as to the revenue of these routes requires an alternative method.

Our method uses the following data:

- Airport statistics as to the number of passengers handled.^{16 17}
- Airport information as to current flight arrivals and departures.¹⁸
- Airliner information as to seating prices, gathered from Hipmunk.¹⁹
 - o Hipmunk is an online price aggregation site. The prices given by Hipmunk are rarely special/opportunistic offers and as such we hope they will be representative of what a typical passenger pays.

The method is as follows:

- Airport statistics were averaged to find a number of passengers per day. Statistics from Auckland Airport were divided into out- and in- bound flights.
- A sample was taken from the international arrivals and departures information, each overseas port was tallied and given a percentage rating.
- Pricing information for flights to and from the local port was found for each overseas port.
- Using the percentage rating, passenger statistics and pricing we were able to calculate the revenue of each route (ie 10% of aircraft fly to Singapore, 10,000 passengers per day at the airport and \$500 a ticket = $10,000 * 0.1 * 500 = \$500,000$ a day).
- Finally the sum of each route gives us the total international revenue generated for the aviation industry by this airport.

(See next page for detailed calculation table).

¹⁶ <http://www.aucklandairport.co.nz/>

¹⁷ <http://www.wellingtonairport.co.nz/>

¹⁸ Author Unknown (2014), "Arrivals and Departures" Retrieved from <http://www.aucklandairport.co.nz/en/FlightInformation/ArrivalsAndDepartures.aspx>

¹⁹ <https://www.hipmunk.com/>

Port	Observed Flights	%	Daily Arrivals	Inbound Price	Inbound Revenue	Daily Departures	Outbound Price	Outbound Revenue
AUCKLAND								
				\$USD	\$USD		\$USD	\$USD
Total	32	100	10400		7615400	9860		8525202.5
Hong Kong	10	31.25	3250	1236	4017000	3081.25	1236	3808425
Sydney	6	18.75	1950	579	1129050	1848.75	579	1070426.25
Singapore	4	12.5	1300	725	942500	1232.5	1784	2198780
Brisbane	4	12.5	1300	305	396500	1232.5	305	375912.5
Melbourne	4	12.5	1300	347	451100	1232.5	347	427677.5
Port Villa	2	6.25	650	480	312000	616.25	480	295800
Nadi	2	6.25	650	565	367250	616.25	565	348181.25
				Totals:				
				Auckland:	16140602.5 USD a day		19000000 NZD/day	
				Wellington:	841770 USD a day		989000 NZD/day	
WELLINGTON								
Port	Observed Flights	%	Daily Pasgrs.	Price:	Revenue:			
				\$USD	\$USD			
Total	14	100	1990		841770			
Melbourne	4	28.57	568.5714286	347	197294.2857			
Brisbane	5	35.71	710.7142857	305	216767.8571			
Nadi	2	14.29	284.2857143	669	190187.1429			
Sydney	3	21.43	426.4285714	557	237520.7143			

Domestic Revenue

Air New Zealand

Air New Zealand does not release revenue figures for domestic services. Once again, we must estimate using other data that they do make publicly available.

ANZ does release data as to revenue passenger kilometres flown (RPK). An RPK indicates one passenger flown 1 kilometre, so a plane carrying 100 passengers over 1000km brings in 100,000RPKs. Therefore, by finding the number of RPKs flown domestically and in total, along with the total revenue, we can find the average revenue per RPK and then estimate the revenue for domestic flights.

Domestic Passenger Kilometres Flown: 5108 million
 Total Flown: 28,059 million
 Total Passenger Revenue: \$3756 million
 Average revenue per RPK: $3756\text{m}/28,059 = \$1.1339/\text{km}$
 Domestic Revenue Estimate: $5108\text{m} * 1.1339 = \584 million/year
 = \$1.87 million a day

Jetstar

Jetstar does not release New Zealand specific data. However, an article released by the NBR claims that they service 22.4% of the domestic market.²⁰ Given the very small size of other carriers we can assume the remaining 77.6% of the market is controlled by Air New Zealand, allowing us to estimate Jetstar's revenue using Air New Zealand figures.

Air New Zealand Revenue: \$1.87 million/day
 Jetstar Revenue: $(\$1.87/0.776)*0.22$
 = \$0.319 million/day

²⁰ <http://www.nbr.co.nz/article/jetstar-nibbles-air-nzs-domestic-dominance-bd-145091>

Total Revenue:

(All figures per day)

Domestic Air NZ:	\$1.87 million
Domestic Jetstar:	\$0.319 million
Auckland Airport Intl:	\$19 million
Wellington Airport Intl:	\$0.989 million
Total:	\$22.178 million a day

Assumptions and limitations:

Our first assumption, which directs the entire problem solving process, is that the eruption of Mount Taranaki was between 5 and 6 on the *Volcanic Explosivity Index*. Based on historic data we believe that a major eruption of around 5 or 6 on the scale is not unlikely to happen. The last major eruption of Taranaki which occurred in 1655 appears to be of a similar magnitude. We can make this assumption if we consider the ambiguity of the original problem.

Secondly, the question asks us to find the cost to the aviation industry. We define cost as loss in revenue. We assume that the loss of revenue to fuel processing companies is negligible due to the proportion of revenue that is derived from New Zealand.

We assume that this eruption takes place under normal conditions. This is a large assumption as it includes factors such as no sporting events & other world events increasing New Zealand aviation traffic, normal climatic conditions, and no rain in surrounding areas.

The main flaw in our mathematical model is that the clumping and accumulation of the ash particles is not taken into account. This means that the ash cloud isn't likely to fly as far as is given in calculations. Ash particles collide and stick together with fluid and ice elements as hydrometeors, and also dry, via electrostatic attraction.²¹

We assume that the cruising altitude of a plane is between 30,000 ft (9.114km) and 45,000 ft (13.716km) and that when volcanic ash particles are no longer in this zone, flights can resume as normal. Furthermore, because different sized particles will fall at different rates we assume that once the upper 75% size of particles (between 7.73 microns and 30 microns) have fallen under the 30,000 ft barrier planes can fly as they can fly through volcanic particles less than 7.73 microns in size.²²

A key limitation is the number sample size (32 & 14) of overseas ports. Due to this low number the sample may not be a true representation of all overseas ports, the important aim of the sample is to find an average ticket price paid to fly into Auckland/Wellington, turning passenger numbers from the airports into revenue figures for the airlines.

The most likely fault would be variations that occur during seasonal changes (for example, flights to business hubs may be more dominant in winter while flights to holiday destinations dominating over summer), which our sample may not cover.

The primary difficulty in improving the sample is obtaining raw information (which could be efficiently parsed using a computer program).

We assume that our selection of Hipmunk prices is in line with what is paid by a typical passenger. It is possible they are not.

²¹ http://www.geo.mtu.edu/~gbluth/Teaching/GE4150/lecture_pdfs/L9a_ashfall.pdf

²² Cheng, D (2003), "Altitude of a commercial jet" Retrieved from <http://hypertextbook.com/facts/2003/DanielCheng.shtml>

Planes are not of a uniform capacity, for example flights from smaller Pacific Island ports may use more small planes, while those to major Asian hubs may use large aircraft. In this scenario our revenue estimates would over represent the Pacific flights (as we assume that numbers of flights equates to passengers carried). This could skew final revenue estimates.

We assume that the revenue from flying RPKs on short domestic flights is the same as revenue from average length flights (including long haul international). Given variations in market competition, efficiency over longer distances, etc, this assumption is unlikely to hold true.

We do not count that internal South Island domestic services are unaffected. We assume such services will have negligible revenues.

It is assumed that airliners will not redirect to Christchurch (thus all ticket revenue is lost). Given Christchurch's isolation from the rest of the country, combined with its small population and damaged infrastructure, it is unlikely that large numbers of international passengers will be redirected there.

Perhaps most critically we assume an eruption of Taranaki will last for only 24 hours, based on past historic data.

Conclusion

Auckland Airport will be closed on a risk basis for Eruption+ 48 hours, it is unlikely that the ash cloud will affect air traffic. Domestic Airspace will reopen after 72 hours (E+ 3.5 days), however it will take a further 24 hours (E+ 4.5 day) for airspace just above Wellington Airport to clear.

Auckland Airport, 1 day:	\$38 million
Wellington Airport, 4.5 days:	$0.989 \times 3.5 = \$3.46$ million
Domestic Airspace 3.5 days:	$(1.87 + 0.319) \times 3.5 = \7.66 million
Total:	\$49.1 million

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