TEAM 1111

If a New Zealand student uploads a video clip that goes viral, how long will it take before 1% of the world's population has seen it?

Introduction/abstract

If a New Zealand student uploads a video clip that goes viral, how long will it take before 1% of the world's population has seen it?

Let us define *viral* as *having an exponential growth phase*. The meaning of the word *viral* stems from the nature of virus propagation[6]; therefore, truly viral phenomena must exhibit exponential growth to qualify as such.

Assuming that this (ordinary, non-celebrity, high school) student uploaded his video to YouTube (it is, after all, certainly the most popular video sharing site in the world)[2], one hundredth of the world's population, at the time of writing, constitutes approximately 70 million people[3]. Supposing—optimistically— that every view represents one unique viewer, this video will need to surpass 70 million views on YouTube.

This is no easy feat. This is *almost twice as many views as Keyboard Cat and Double Rainbow.*

Without introducing any mathematical model at all, it is common knowledge that YouTube view rates decline over time after having passed a certain point in their lifetimes. It is also common knowledge that New Zealand content finds it very difficult to achieve world-recognition status. As such, the answer to the question *when* could easily be 'never', in the reasonable future. In the following section, we will introduce the model to demonstrate this fact.

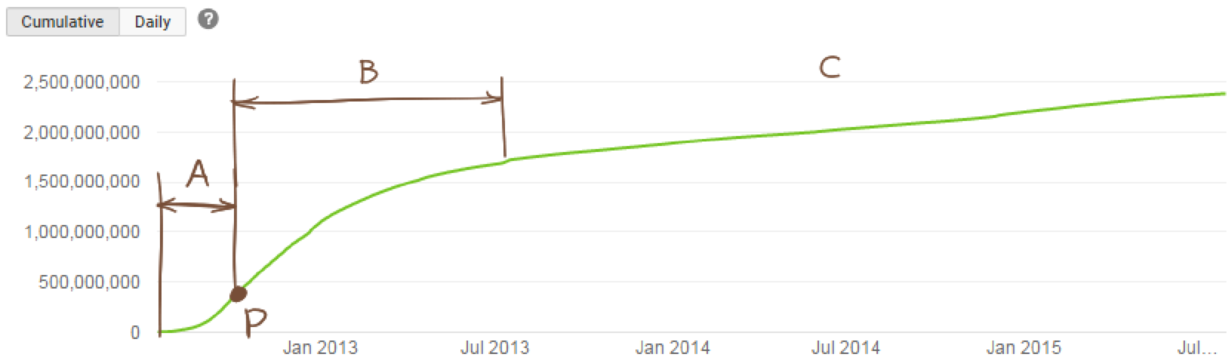
Process & method

- Explain the approach used to arrive at the final conclusion, in a formal and organized fashion.
- Mention any assumptions made during the process and explain how they might affect the conclusion.
- Answer the original question.

Analysis

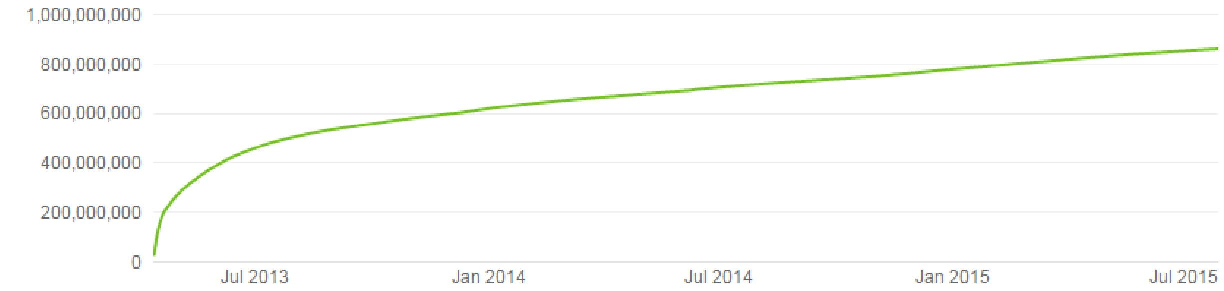
We may begin our inspection with an examination of the cumulative-view graphs of known viral videos.

Gangnam Style [1]



This is one of the most classic viral videos in YouTube history. By observing its graph, we may draw some conclusions about the nature of the propagation of viral videos. Section A represents the exponential curve section of the graph, the *viral* section; point P is a point of inflection, and thereafter the graph resembles a logarithmic curve (section B) for a time as its growth slows, before it takes on a roughly linear view aggregation pattern (section C). We notice, first of all, that the exponential phase is brief in comparison to the stable phases, and that the stable phases account for the bulk of the views that the video eventually achieves. The exponential phase, however, is unique to *viral* videos: videos like *Gentleman,* Psy's second popular YouTube song, has only the stable curve phases.

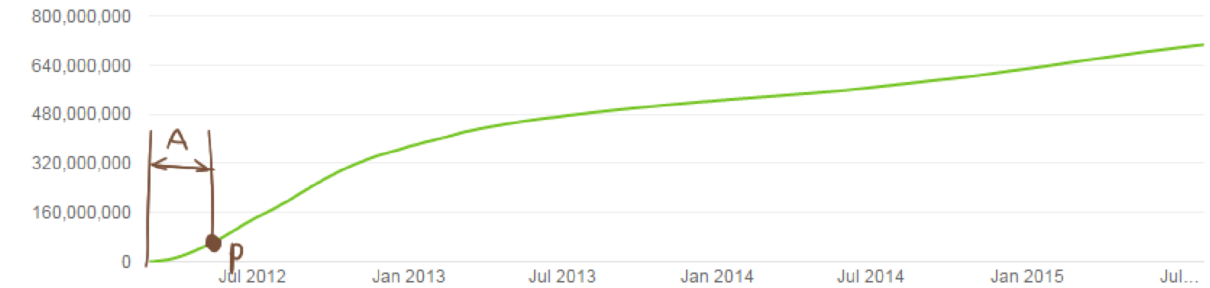
Gentleman[4]



The logarithmic-then-linear stable curve seems to be common to many 'non-viral popular' YouTube videos, which we may define as videos that achieve high view counts largely due to an existing fanbase or initial hype. *Viral* propagation—propagation that produces an exponential growth curve in terms of views—only occurs when the fanbase of a particular artist or video is massively and rapidly expanded through channels such as sharing and media attention.

Here is an additional view graph for a 'truly viral' video:

Call Me Maybe[5]



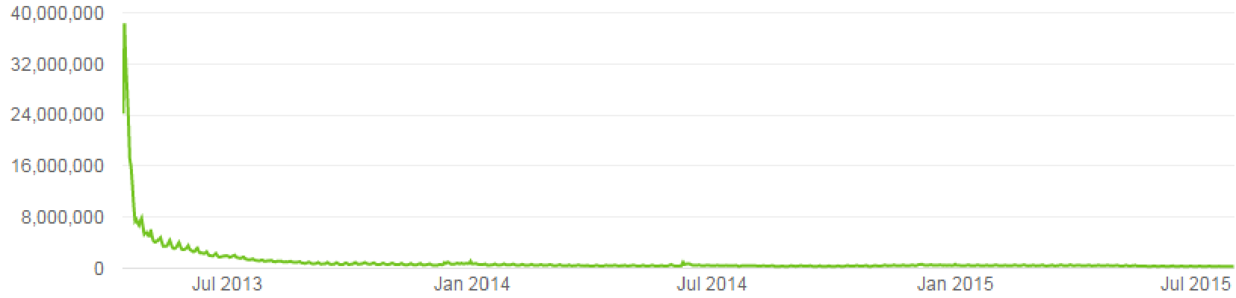
Both *Gangnam Style* and *Call Me Maybe* are characterized by a sharp initial climb in daily views to a single peak, as shown. This generates the exponential propagation that can be seen in the cumulative view graphs.

Gangnam Style (daily views) [2]

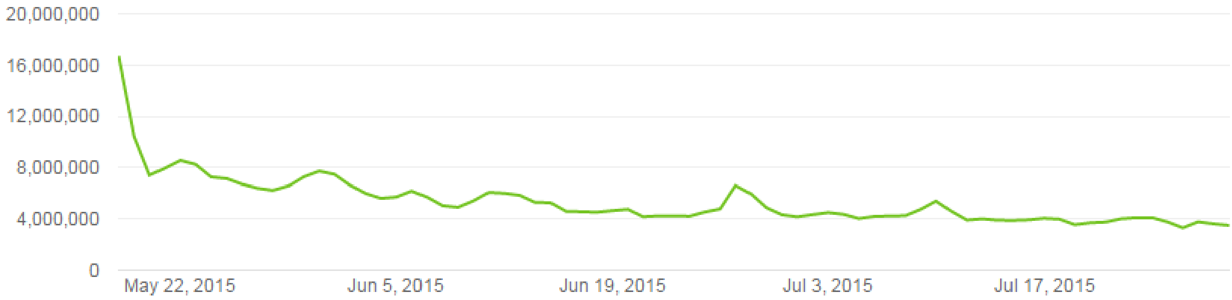
Call Me Maybe (daily views) [5]

On the contrary, 'non-viral popular' videos tend to exhibit either a peak almost immediately after posting, as the daily view graphs for *Gentleman* and Taylor Swift's *Bad Blood* demonstrate:

Gentleman *[4]*



Bad Blood *[7]*



Modelling

Having made these observations, we may model the cumulative view graphs of truly viral videos as follows:

$0 \leq t \leq P$: *exponential growth*
$P \leq t < T$: *logarithmic growth*
$t \geq T$: *linear growth* where t stands for time after posting.

Mathematically, this translates to a piecewise function. To generate a smooth curve with manipulatable variables, we produced the following function*:

$$v(t) = \begin{cases} f(t) = ke^{r(t-P)} - ke^{-rP}, \quad 0 \leq t \leq P \\ g(t) = \dfrac{rk}{q}\ln(q(t-P)+1) + k - ke^{-rP}, \quad P \leq t < T \\ l(t) = g'(T)t + [g(T) - Tg'(T)], \quad t \geq T \end{cases}$$

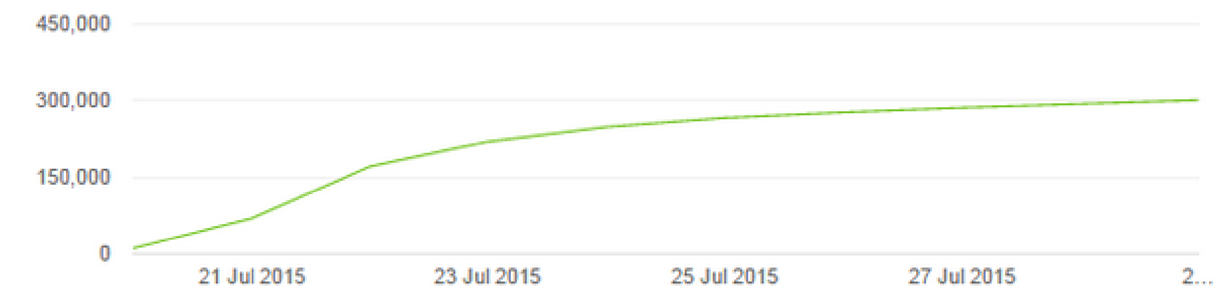
*Refer to sub-section *Derivation* for derivation process.

This function has 4 variables, other than *t,* which need to be calibrated.

P, the point of inflection.
r, k and *q,* variables which control the horizontal and vertical stretch factors of both graphs.
T, the point at which linear growth begins.

In real life, these variables represent the share rates of the videos, as well as how long viral fame generally endures before the video settles into a generally slower and more stable mode of view accumulation. These variables can be manipulated so as to represent any truly viral video's growth curve. In order to calibrate them so that they accurately represent market conditions in New Zealand, we can match them to the view graph taken a video uploaded by a New Zealand student that has recently gone 'viral' [9]:

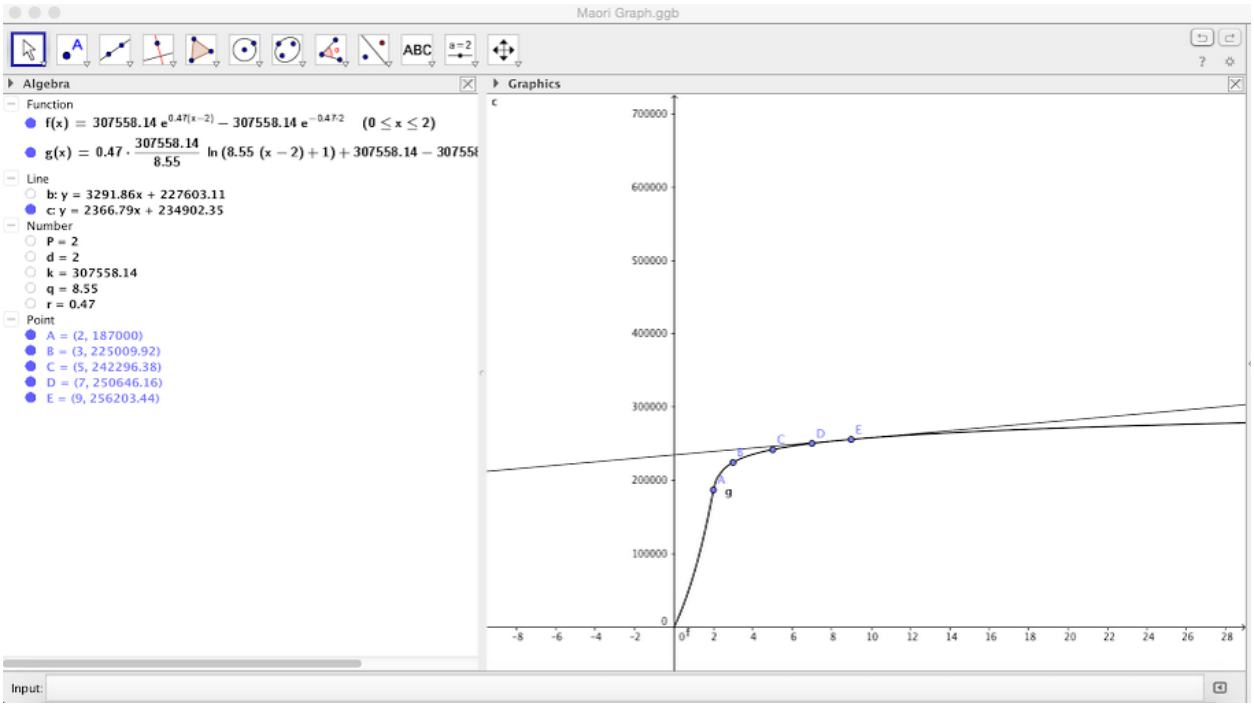
"The importance of correctly pronouncing Maori words": cumulative view graph *[8]*



"The importance of correctly pronouncing Maori words": selected points

Video Name	1	2	3	4	5	7
Number of views The importance of correctly pronouncing	72000	**18700 (a)**	225000		263000	281750

*"The importance of correctly pronouncing Maori words": MODEL**



$P = 2$
$k = 307558.14$
$q = 8.55$
$r = 0.47$

*Refer to sub-section *Derivation* for derivation process.

With this model, it takes approximately **81 years** to achieve 70 million views. YouTube has only existed for 10 years; we cannot guarantee the accuracy of our model, or really any model, beyond that timeframe. Therefore, for all practical intents and purposes, this video will never achieve viewing by 1% of the world's population.

Derivation

$$v(t) = \begin{cases} f(t) = ke^{r(t-P)} - ke^{-rP}, & 0 \le t \le P \\ g(t) = \frac{rk}{q}\ln(q(t-P)+1) + k - ke^{-rP}, & P \le t < \\ l(t) = g'(T)t + [g(T) - Tg'(T)], & t \ge T \end{cases}$$

We shall now attempt to explain the processes that we took to derive these equations.

Firstly, we assumed that section a of the trend followed the equation, $f(t) = ke^{r(t-P)} + c_1,\ 0 \le t \le P$ where c_1, is a constant, and the section b of the trend to be

$g(t) = s\log(q(t-P)+1) + c_2,\ P \le t < T$ where c_2 is another constant, contributing to g's horizontal shift, and s is the vertical stretch of the graph g. Since the graph is continuous, $f(P) = g(P)$, and thus $f'(P) = g'(P)$. We can use these two simple rules to determine the two constants.

(You may notice for this $g(t)$, the equation involves log, but our final equation is in natural logarithm, and you shall see that the base a or 10 log turns into a natural log with appropriate simplification.)

$$t = P$$
$$f(P) = ke^{r(P-P)} + c_1$$
$$f(P) = k + c_1$$
$$g(P) = s\log(q(P-P)+1) + c_2$$
$$g(P) = s\log(1) + c_2$$
$$g(P) = c_2$$
$$f(P) = g(P)$$
$$k + c_1 = c_2$$

We shall now sensibly assume that when $t = 0$, $f(t) = 0$, as it makes sense that there'll be no views at the instant the video is uploaded.

$$f(0) = ke^{r(-P)} + c_1$$
$$f(0) = ke^{-rP} + c_1 = 0$$
$$c_1 = -ke^{-rP}$$
$$\therefore c_2 = k - ke^{-rP}$$

Now we shall now use $f'(P) = g'(P)$ to work our s, c_1, and c_2:

$$f'(t) = ke^{r(t-P)} \times \frac{d}{dt}\big(r(t-p)\big)$$
$$= ke^{r(x-P)} \times r$$
$$= rk\, e^{r(x-P)}$$
$$\therefore f'(P) = rk$$
$$g'(t) = s \times \frac{d}{dt}[\log(q(t-P)+1) + c_2]$$
$$g'(t) = s \times \frac{q}{(q(t-P)+1)ln(10)}$$
$$g'(t) = \frac{sq}{(q(t-P)+1)ln(10)}$$
$$g'(P) = \frac{sq}{ln(10)}$$
$$Applying\ f'(P) = g'(P):$$
$$rk = \frac{sq}{\ln(10)}$$
$$\therefore s = \frac{rk}{q}ln(10)$$
$$\therefore g(t) = \frac{rk}{q}\ln(10) \times \log(q(t-P)+1) + c_2$$
$$g(t) = \frac{rk}{q}\frac{\log(10)}{\log(e)} \times \frac{\log(q(t-P)+1)}{\log(10)} + c_2$$
$$\log(10)\ cancels\ out$$
$$\therefore g(t) = \frac{rk}{q}ln(q(t-P)+1) + c_2$$

Finally, substituting our two constant values, our equations are:

$$f(t) = ke^{r(t-P)} - ke^{-rP}, 0 \le t \le P$$
$$g(t) = \frac{rk}{q}\ln(q(t-P)+1) + k - ke^{-rP}, P \le t < T$$

Now we shall derive the equation for the linear function that we assume the views to grow at a constant rate or at the same rate of change / tangenet of the logarithimic graph at a chosen point $t = T$ where we believe that the graph acheives a constant rate of growth.

Equation of a linear equation:

$The\ point\ we\ assume\ to\ have\ a\ constant\ growth\ is\ T,$
$the\ gradient\ is\ m, the\ constant\ (horizontal\ shift)\ is\ c$

$$l(t) = mt + c$$
$$m = g'(T)$$
$$At\ l(T) = g'(T) \times T + c = g(T)$$
$$c = g(T) - g'(T) \times T$$
$$l(t) = g'(T)t + [g(T) - Tg'(T)], t \ge T$$

Now we shall apply these equations to our example of the Maori video data.

We shall let t, be time in days, and so we know the points

$(1{,}7200), (2{,}187500), (3{,}225000), (5{,}263000), and\ (7, 2$

We shall assume $P = 2$, as it looks reasonable for the concavity to change at that point.

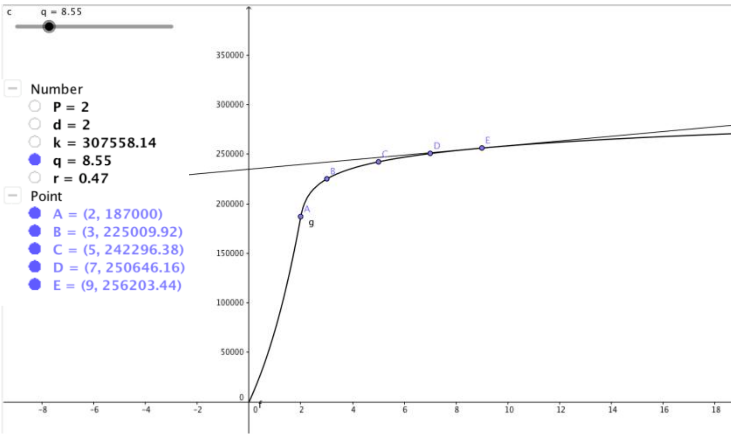
$$(P = 2, y_P = 187000)$$
$$187000 = ke^{r(2-2)} - ke^{-r\times 2}$$
$$k - ke^{-2r} = 187000$$
$$k(1 - e^{-2r}) = 187000\ \ [1]$$
$$(1{,}7200)$$
$$72000 = ke^{r(1-2)} - ke^{-r\times 2}$$
$$72000 = ke^{-r} - ke^{-2r}$$
$$ke^{-r}(1 - e^{-r}) = 72000\ \ [2]$$
$$\frac{[2]}{[1]}$$
$$\frac{ke^{-r}(1-e^{-r})}{k(1-e^{-2r})} = \frac{72000}{187000}$$
$$\frac{e^{-r}(1-e^{-r})}{(1-e^{-r})(1+e^{-r})} = \frac{72}{187}$$

$$\frac{e^{-r}}{(1+e^{-r})} = \frac{72}{187}$$
$$e^{-r} = \frac{72}{187} + \frac{72}{187}e^{-r}$$
$$e^{r} = \frac{115}{72}$$
$$r = \ln\left(\frac{115}{72}\right) = 0.468266 \rightarrow (A)$$
$$k = \frac{187000}{(1-e^{-2r})}$$
$$k = 307558.1396 \rightarrow (B)$$

We shall now work out the value of q of the log graph.

$$(3{,}225000)$$
$$225000 = \frac{rk}{q}\ln(q(3-2)+1) + k - ke^{-2r}$$
$$225000 = \frac{rk}{q}\ln(q+1) + k + ke^{-2r}$$
$$225000 - k - ke^{-2r} = \frac{rk}{q}\ln(q+1)$$
$$e^{\frac{225000-k-ke^{-2r}}{rk}} = (q+1)^{\frac{1}{q}}$$
$$\therefore e^{\frac{225000-(B)-(B)e^{-2(A)}}{(A)(B)}} = (q+1)^{\frac{1}{q}}$$

We can clearly observe that calculating the value for q is extremely tedious and complicated. Instead, we used the tools of GeoGebra to input our known constants of $P, r, k,$ *and both time and view values* to calculate the value of q, we would need to work out (3,225000). Basically, we adjusted the value of q, using a slider, to work out what q needs to be for the



graph to approximately have a value of 225000 at $t = 3$.
Therefore, now we know that approximately, $q = 8.55$ for this example of the model.
Finally, we shall assume that $T = 9$ to be our point where we assume that the growth of the views goes to a constant rate.

$$\therefore g(9) = 256203.44$$

We know that

$$g'(t) = \frac{sq}{(q(t-P)+1)\ln(10)}$$
$$g'(t) = \frac{\frac{rk}{q}\ln(10) \times q}{(q(t-P)+1)\ln(10)}$$
$$g'(t) = \frac{rk}{(q(t-P)+1)}$$
$$\therefore g'(t) = \frac{rk}{(q(t-2)+1)}$$
$$\therefore g'(9) = \frac{(A)(B)}{(8.55(9-2)+1)}$$
$$g'(9) = 2366.788$$

We know that $l(t) = g'(T)t + [g(T) - Tg'(T)], \quad t \geq T$

$$l(t) = 2366.788\,t \;+\; [256203.44 - 9 \times (2366.788)]$$
$$l(t) = 2366.788t + 234902.352$$

Now we need to work out the number of days required to reach 1% of world's population views, which is 70million views, therefore $l(t) = 70{,}000{,}000$

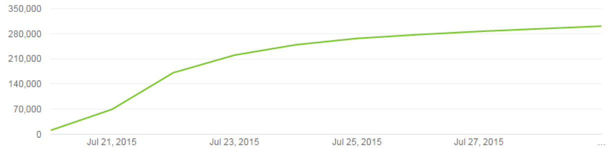
$$70{,}000{,}000 = 2366.788t + 234902.352$$
$$2366.788t = 697656097.65$$
$$t = 29476.705\ days$$
$$t = 29476.705\ days \times \frac{1\ year}{365.25\ days}$$
$$t = 80.7\ years$$

Assumptions

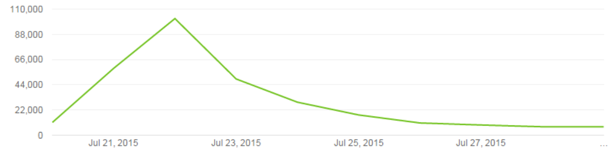
World population
For the sake of simplicity, we assumed that the population of the world remains constant. Obviously, this affects the accuracy of our results; however, seeing as our figure of 70 million was approximate to begin with—different estimations by different organizations yield different figures [3]— and seeing as population projections for the year 2150 made by qualified organizations vary by over 20 billion [3], the decision was made to assume a constant population to avoid headaches.

This video is representative of New Zealand produced YouTube videos
We selected a single video that we believed to be relatively representative of the content that an average New Zealand student might produce. The video's graphs qualified it as a 'viral video' by our definitions, displaying both the A-B-C structure for the cumulative graph and the climb to a sharp peak for the daily graph.

Cumulative



Daily



However, viewing statistics will naturally vary greatly—extremely greatly—between different videos. The tastes of the online community are notoriously fickle, and choosing a different video might well have altered our answer by orders of magnitude. (After all, is the Oscar-winning Lorde not a New Zealand student?) Had we had more time, we would have analysed several videos produced by New Zealand students, not only one.

The linear trend continues indefinitely
The answer of 81 years is in fact highly arbitrary and not so much an accurate prediction as a demonstration that, for all practical intents and purposes, this video will never achieve 70 million views. As mentioned earlier, YouTube itself has existed for only 10 years, and so even if the oldest YouTube videos in existence were analysed, it would be impossible to produce a model of aggregate view count that would last for 81 years based on real life.

Approximation in a different situation
We recognize that our method of analysing one video is somewhat reductionist. While we will not have the time to construct a different model for an entirely new situation, we can make some approximations.

Lorde is also a New Zealand student, albeit a drastically different one from the young man who produced 'The Importance of Correctly Pronouncing Maori Words': she has won Oscars and become a celebrity. Her own YouTube channel boasts two videos, *Team* and *Royal*. By looking at her channel and the projection from http://socialblade.com/youtube/user/lordemusic/futureprojections/views , we can predict the time when Lorde's Royal will reach 73 million (1%) of the world's population.

However because the site predicts the whole channel views, we need to make some assumptions in order to proceed with the calculate.

A major assumption that we are making is that currently the channel has a total of 85,162,951 views as at 1/08/2015 and Royal takes up 72.39% of the total number of views. We are going to assume that the projections from the website will continue to follow this trend, 73.39% of the projection increase will go towards the views for Royals and the rest will go towards the views for Team.

Currently:

Total views: 85,162,951
Views on Royal: 61,652,917

In 30/07/2018- Projection from http://socialblade.com/youtube/user/lordemusic/futureprojections/views .

Projected views: 101,621,000
Difference: 101,621,000 - 85,162,951 = 16458049

Projection increase to Royals: 16458049 x 0.7239 = 11913982

Projection total of Royals on 30/07/2018= 11913982 + 61652917 = 73,566,899 ≈ 73.6 million.

Time taken for Royals to reach 1% of world population:

Released: 12/05/ 2013
Projection around 30/07/2018
Duration: 1905 days roughly. or 5 years 2 months and 18 days.

Conclusion

Our model indicates that, practically speaking, a YouTube video uploaded by a New Zealand student—even one that achieves the viral growth curve, within our modest community—will effectively never achieve viewing by 1% of the world's population. Had we had more time to complete the task, we would have investigated a number of videos to produce a holistic picture of the New Zealand YouTube community; we recognize that to base an entire model off one video is highly reductionist.

We would also have investigated older YouTube videos in order to gain an idea of the trends that aggregate view graphs will fall into after a much longer period of time than any of the more recent viral videos have had to propagate. Realistically speaking, it is unlikely that a video's views will remain in linear growth indefinitely; they will more likely lapse into a second phase of logarithmic growth. However, because our existing model clearly demonstrated that expecting the video to attain 70 million views was unrealistic, regardless of end behaviour, we decided that further modifying our model to accurately reflect end behaviour would be of little purpose in the context of this question.

In conclusion, we can state that New Zealand students would need a little more clout on the world stage if they wanted to achieve 70 million views for their YouTube videos.

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